Toppers Village.

Class - 11 Chapter - 2

Relations and Functions

Exercise 2.1 - Basics

Sets. ✓
↓
ordered Pairs
↓
C.P.
↓
Rel.
↓
Fn.

Ordered Pairs : (x, y)

2 elements in Particular order

(x, y) $(2, 3)$ $(5, -3)$ $(3, 2)$

$(2,3) \neq (3,2)$

$x \in \text{Set}_1$

$y \in \text{set}_2$

set $= \{2, 3\} = \{3, 2\}$

Prop. of Ordered Pairs.

(I) $(x, y) = (a, b)$ → $x = a$, $y = b$

ex. If $(x+3, y-1) = (7, 8)$, then find

$x \longrightarrow x+3=7 \Rightarrow x=4$

$y \longrightarrow y-1=8 \Rightarrow y=9$

Cartesian Product of Sets (A & B) $= A \times B$
$\neq B \times A$

A = set (non empty)
B = set (non empty)

$A \times B$ = Set of all possible ordered Pairs from A to B.

$= \{(x, y) : x \in A \text{ and } y \in B\}$

$B \times A$ = set of all possible ordered pairs from B to A

$B \times A = \{(x, y) : x \in B, y \in A\}$

Examples.

A = { Red, yellow } ← Colors

B = { Pants, Shirt } ← Apprals

$A \times B$ = { (Color, App.), , }

this type

$A \times B$ = { (Red, Pants), (red, Shirt), (yellow, pant), (yellow, Shirt) } eg:

$A = \{1, 2, 3\}$

$B = \{a, b, c\}$

	a	b	c
1 →	(1,a)	(1,b)	(1,c)
2	(2,a)	(2,b)	(2,c)
3	(3,a)	(3,b)	(3,c)

↑

$A \times B$

$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a), (3,b), (3,c)\}$

$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3)\}$

$B \times A = \{(x, y) : x \in B, y \in A\}$

a, b, c   1, 2, 3

In above example

$n(A) = 3$

$n(B) = 3$

$n(A \times B) = 9 = n(B \times A)$

Properties of ~~orded~~ ordered Pairs.

(I) $n(A) = n_1$, $n(B) = n_2$ $A \times B \neq B \times A$

$$\boxed{n(A \times B) = n(B \times A) = \text{no. of elements} = n_1 \cdot n_2}$$

(II) If Either A or B is empty set then $A \times B = \phi = \{\ \}$

$A = \{\ \}$ $B = \{1, 2, 3\}$

$\times$ (,1) (,2) (,3)

Doesnot make any sense.

(III) If either A or B is infinite set then $A \times B$ is also an infinite set.

$A = \{2\}$ $B = \{1, 2, 3 \ldots\}$

$A \times B = \{(2,1), (2,2), (2,3), (2,4), (2,5) \ldots\}$

2   1, 2, 3 ...

(IV) $A \times A = \{(x, y),\ x \in A,\ y \in A\}$

e.g. $A = \{1, 2\} \longrightarrow n(A) = 2$

$A = \{1, 2\} \longrightarrow n(B) = 2$ $\quad \underline{n(A \times B) = 4}$

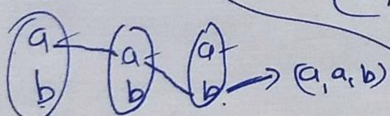
$A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$

(V) $A \times A \times A$ = set of ordered triplets

$= \{(x, y, z) : x \in A, y \in A, z \in A\}$

e.g. ~~$A = \{(a, b)\}$~~ $A = \{a, b\}$

$A \times A \times A = \{(a,a,a), (a,a,b), (a,b,a),$
$(a,b,b), (b,a,a), (b,a,b)$
$(b,b,a), (b,b,b)\}$



(IV) $A \times A = \{(x, y),\ x \in A,\ y \in A\}$

e.g. $A = \{1, 2\} \longrightarrow n(A) = 2$

$A = \{1, 2\} \longrightarrow n(B) = 2$ $\quad n(A \times B) = 4$

$A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$

(V) $A \times A \times A$ = set of ordered triplets

$= \{(x, y, z) : x \in A, y \in A, z \in A\}$

e.g. ~~A = {(a, b)}~~ $A = \{a, b\}$

$A \times A \times A = \{(a,a,a), (a,a,b), (a,b,a), (a,b,b), (b,a,a), (b,a,b), (b,b,a), (b,b,b)\}$

$\begin{matrix} a \\ b \end{matrix} \ \begin{matrix} a \\ b \end{matrix} \ \begin{matrix} a \\ b \end{matrix} \rightarrow (a,a,b)$

Toppers Village:

Exercise 2.1

(1) $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$

$\frac{x}{3} + 1 = \frac{5}{3} \Rightarrow \frac{x}{3} = \frac{5}{3} - 1 = \frac{2}{3} \Rightarrow x = 2$

$y - \frac{2}{3} = \frac{1}{3} \Rightarrow y = \frac{2}{3} + \frac{1}{3} \Rightarrow y = 1$

(2) $n(A \times B) = n(A) \times n(B)$

$= 3 \times 3 = 9$ elements

(3) $G = \{7, 8\}$ $\quad H = \{5, 4, 2\}$

$G \times H = \{(7,5), (7,4), (7,2), (8,5), (8,4), (8,2)\}$

6

$H \times G = \{(5,7), (4,7), (2,7), (5,8), (4,8), (2,8)\}$

(4) ~~A~~ $P = \{m, n\}$

(I) F $Q = \{n, m\} = \{m, n\}$

$P \times Q = \{(m,n), (m,m), (n,n), (n,m)\}$

(II) T

(III) $LHS = A \times (B \cap \phi)$ $\{3,4\} \cap \{\ \}$

$= A \times (\phi)$ Empty

$= \phi$

$= RHS$

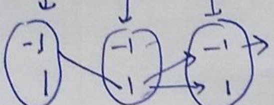
↓

(T)

(5) $A = \{-1, 1\}$

ordered triplet

$A \times A \times A = \{(-1,-1,-1), (-1,-1,1), (-1,1,-1), (-1,1,1), (1,-1,-1), (1,-1,1), (1,1,-1), (1,1,1)\}$



(6) $A \times B = \{(a,x), (a,y), (b,x), (b,y)\}$

~~A = {~~ $A = ? = \{a, b\}$

$B = ? = \{x, y\}$

(7) $A = \{1,2\}$ $B = \{1,2,3,4\}$

$C = \{5,6\}$ $D = \{5,6,7,8\}$

(i) $\boxed{A \times (B \cap C) = (A \times B) \cap (A \times C)}$

$B \cap C = \phi$

$LHS = A \times (B \cap C)$

$= A \times (\phi)$

$= A \times \phi = \phi$

~~RHS =~~

$A \times B = \{(1,1)(1,2),(1,3)(1,4),(2,1)(2,2)(2,3)(2,4)\}$

$A \times C = \{(1,5),(1,6),(2,5),(2,6)\}$

$RHS = (A \times B) \cap (A \times C)$

$= \phi = LHS.$ ✓

(ii) $A \times C$ is a subset of $B \times D$

Prove.

1
2
3
4

5
6
7
8

$A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

$B \times D = \{(1,5), (1,6), (2,5), (2,6), (1,7), (1,8), (2,7), (2,8), (3,5) \ldots\}$.

Clearly $A \times C \subset B \times D$

(8) $A = \{1,2\}$

$B = \{3,4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

no. of elements in $A \times B = 4 = n$

no. of subset of $A \times B = 2^n = 2^4$

$= 16$

All subsets of $A \times B \Rightarrow$

ϕ

$\{(1,3)\}$

$\{(1,4)\}$

$\{(2,3)\}$

$\{(2,4)\}$

$\{(1,3), (1,4)\}$

$\{(1,3), (2,3)\}$

$\{(1,3), (2,4)\}$

$\{(1,4), (2,3)\}$

$\{(1,4), (2,4)\}$

$\{(2,3), (2,4)\}$

$\{(1,3), (1,4), (2,4)\}$

$\{(1,3), (1,4), (2,3)\}$

$\{(1,3), (2,3), (2,4)\}$

$\{(1,4), (2,3), (2,4)\}$

$\{(1,3), (1,4), (2,3), (2,4)\}$

16

(9) $n(A) = 3$.

$n(B) = 2$

$(x, 1), (y, 2), (z, 1) \in A \times B$

$A = \{x, y, z\}$

$B = \{1, 2\}$

---

(10) no. of elements in A = n = (+)

$n(A \times A) = n \times n = n^2 = 9$

$\Rightarrow n^2 = 9$

$n = \pm 3$

$\boxed{n = 3}$

no. of elements in A = $n = 3$

$(-1, 0), (0, 1) \in A \times A$

$A = \{-1, 0, 1\}$

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Basics of Exercise $\boxed{2.2}$

Relations and Functions. — Chapter 2

Relation:

A relation (R) from a non-empty set A to a non-empty set B is a (subset) of Cartesian product $A \times B$.

Sets.
↓
ordered Pairs
↓
C.P. $(A \times B)$
↓
Rel^n.
↓
Fn^n.

Set Builder Form:

$$R = \{(x,y) : \text{Relation (equation) in } x \,\&\, y\}$$

$x \in A$, $y \in B$

↓

R from A to B / ARB / xRy /

$R : A \rightarrow B$

Here image of $x = y$

Preimage of $y = x$

Example.

Set A = { Amit B., ~~Aamir~~ Aamir K., Salman K., Akshay K. }

↑
actors.

Set B = { Baghban, PK, Singham, 3 I }

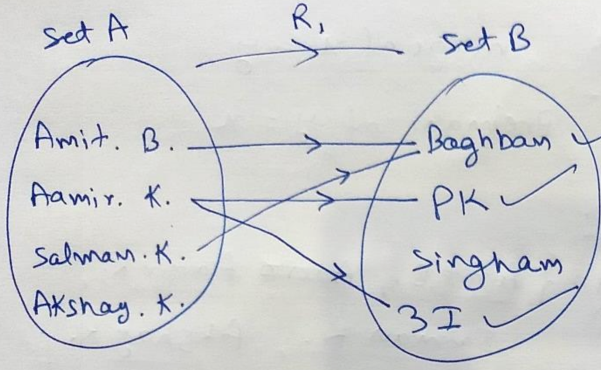
Cartesian product $(A \rightarrow B)$ $4 \times 4 = 16$

$A \times B = \{$ All ordered Pairs $\}$
↓
total 16.

$R_1 = \{(x,y) : \text{'}x\text{' acted in movie '}y\text{'}\}$

$R_1 : A \rightarrow B$

↑
Set Builder



Arrow Diagram. $R_1 : A \to B$

Roster form:

$R_1 = \{$ (AB, Baghban), (Aa.K, PK), (Aa.K, 3I), (Salman.K, Baghban) $\}$

image of AB = Baghban

image of Aa.K. = PK, 3I

$R_1 : X \to Y$

Pre-image of Baghban = Amit B, Salman K.

Pre-image of PK = Aamir K.

Domain: set of first elements (Connected) in any Relation.

Domain = { AB, Aa.K, Salman.K }

Range: Set of second elements (connected) in any relation.

~~RB~~ Range = { Baghban, PK, 3I }.

Codomain: the complete set B

$R_1 : A \to B$ → the second set.

Codomain = { Baghban, PK, Singham, 3I }

Note: Range $\subseteq$ Codomain

Range $\downarrow$ set of only Connected elements of B

Codomain $\downarrow$ Complete set B

Note: R from A to B : $R: A \to B$

$R: A \to A$ = R from A to A

= R on A

Example: $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : x = y + 2\}$ defined on A.

$x \in A$ $\quad y \in A$ $\leftarrow$ $R: A \to A$

$x = y + 2$ $\quad y \in A$

$y = 1$ $\quad x = 3 \in A$ ✓

$y = 2$ $\quad x = 4 \in A$ ✓

$y = 3$, $x = 5 \in A$ ✓

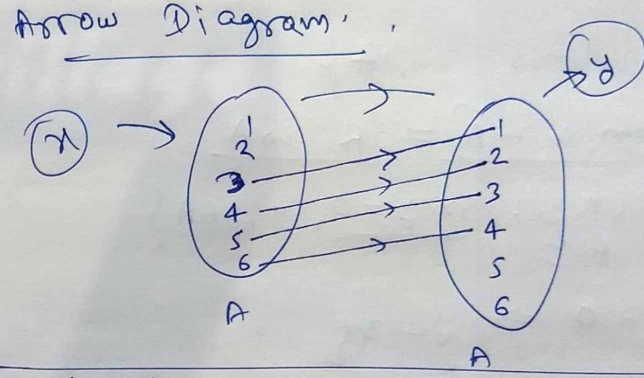
$y = 4$, $x = 6 \in A$ ✓

$y = 5$, $x = 7 \notin A$

$R = \{(3,1), (4,2), (5,3), (6,4)\}$

Roster form

Arrow Diagram:



No. of Relation from A to B = no. of subsets of Cartesian product $A \times B$

$= 2^{p.q}$

$n(A) = p$

$n(B) = q$ $\qquad 2^n$

$n(A \times B) = pq$

E.g. $A = \{2, 3, 4\}$, $B = \{a, b\}$

No. of relations from A to B $= 2^{3 \times 2} = 2^6 = 64$

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Exercise : 2.2 (All questions)

① $A = \{1, 2, 3, \ldots, 14\}$

R from A to A = R on A

$R = \{(x, y) : 3x - y = 0,\ x, y \in A\}$

$3x - y = 0 \Rightarrow y = 3x$

$y \in A$ $\quad$ $x \in A$

x	1	2	3	4	~~5~~
$y = 3x$	3	6	9	12	~~15~~

→ not possible.

$R = \{(1,3), (2,6), (3,9), (4,12)\}$

Domain $= \{1, 2, 3, 4\}$

Range $= \{3, 6, 9, 12\}$

Codomain $= \{1, 2, 3, \ldots 14\}$ = A

② $R : N \to N$ = R on N

$R = \{(x, y) : y = x + 5\}$

$x \in N$ $\quad$ x is less than 4

$x = 1, 2, 3$

$y = 6, 7, 8$

$R = \{(1,6), (2,7), (3,8)\}$

Domain $= \{1, 2, 3\}$

Range $= \{6, 7, 8\}$

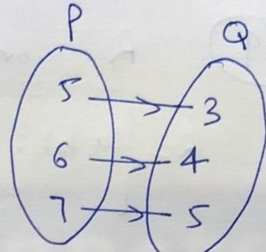
③ $A = \{1, 2, 3, 5\}$

$B = \{4, 6, 9\}$

Diff. = odd = odd − Even → Even − odd

$R = \{(1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)\}$

(4)



(i) Set Builder Form:

$R = \{(x, y) : x - 2 = y,\ x \in P,\ y \in Q\}$

(ii) Roster Form:

$R = \{(5,3), (6,4), (7,5)\}$

Domain $= \{5, 6, 7\}$

Range $= \{3, 4, 5\}$

(5) $A = \{1, 2, 3, 4, 6\}$ $\quad \frac{b}{a}$

$R = \{(a, b) : a, b \in A,\ b \text{ is exactly divisible by } a\}$

$\frac{b}{a}$ $\quad \frac{b}{1}$ $\quad \frac{2}{1}$ ✓ $\quad \cancel{\frac{6}{4}}$ $\quad \frac{1}{2}$ $\quad \cancel{\frac{6}{3}}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)\}$

Domain $= \{1, 2, 3, 4, 6\}$

Range $= \{1, 2, 3, 4, 6\}$

(6) $R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

$R = \{(0,5), (1,6), (2,7), (3,8), (4,9), (5,10)\}$

Domain $= \{0, 1, 2, 3, 4, 5\}$

Range $= \{5, 6, 7, 8, 9, 10\}$

(7) $R = \{(x, x^3) : x \text{ is prime no. less than } 10\}$

$x = 2, 3, 5, 7,$

$R = \{(2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3)\}$

$= \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

(8) $A = \{x, y, z\} \longrightarrow n(A) = 3 = p$

$B = \{1, 2\} \longrightarrow n(B) = 2 = q$

no. of relations from A to B

$= 2^{p.q}$

$= 2^{3\times 2}$

$= 2^6 = 64$

(9) R on z

$R = \{(a, b) : a, b \in z, a - b \text{ is an integer}\}$

set of integers

$\boxed{\text{Int}_1 - \text{Int}_2 = \text{Int.}}$

$a \in z$

$b \in z$

Domain = Integers = z

Range = z

Toppers Village:

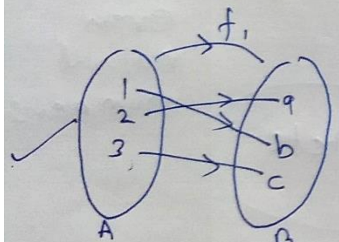
Class - 11 Maths

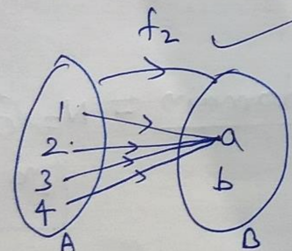
Sets
↓
ordered Pairs
↓
$A \times B$
↓
Rel^n
↓
Fn^n

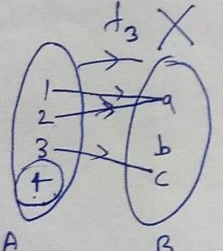
Functions | Map | Mappings:

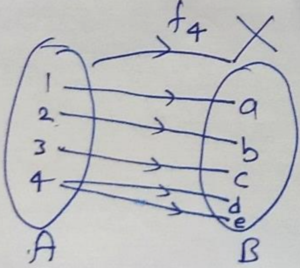
Definition (I): $\boxed{f: A \to B}$

A relation 'f' from a set 'A' to a set 'B' is known as function if each element of 'A' has unique (one and only one) image in 'B'.









✓ $f_1 = \{(1,b), (2,a), (3,c)\}$

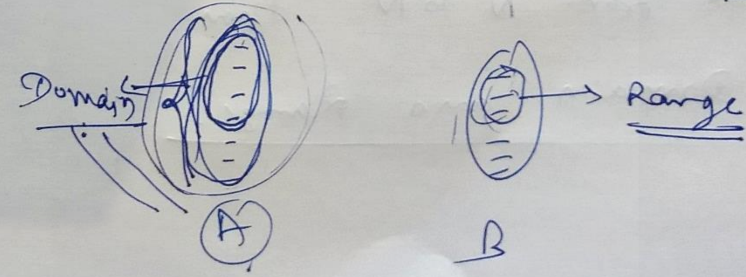
✓ $f_2 = \{(1,a), (2,a), (3,a), (4,a)\}$

✗ $f_3 = \{(1,a), (2,a), (3,c)\}$ $\cancel{(4,\ )}$ $A = \{1,2,3,4\}$, $B = \{a,b,c\}$

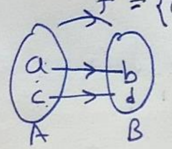
✗ $f_4 = \{(1,a), (2,b), (3,c), (4,d), (4,e)\}$

Definition - (II)

In other words, a function f from A to B such that the domain of 'f' is $\boxed{A}$ and no two distinct ordered pairs in f have the same first element.



Note: $f : A \to B$



$f = \{(a, b), (c, d)\}$

(I) If 'f' is a function from A to B and $(a, b) \in f$

then $\boxed{f(a) = b}$

$\cancel{P(x) = x^2 + 3}$

$\cancel{P(1) = 1^2 + 3}$

image of 'a' under 'f' = b

image of 'c' under 'f' = d

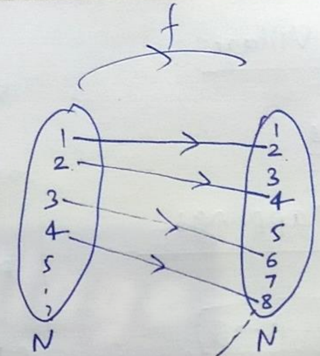
preimage of 'b' under 'f' = 'a'

Preimage of 'd' under 'f' = c

---

e.g. funⁿ $f = \{(x, y) : y = 2x,\ x, y \in N\}$ (on N)

defined from N to N then find Domain and Range:



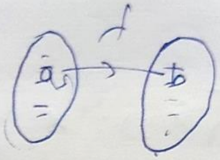
x $\boxed{y = 2x}$ ☆

Domain = $\{1, 2, 3, 4, 5, 6, \ldots\} = N$

Range = $\{2, 4, 6, 8, \ldots\}$ = set of even natural numbers.

Codomain = N = Complete 2nd set

Real valued function:



those functions whose range is 'R' (real numbers) or subset of 'R'.

$f(a) = b$

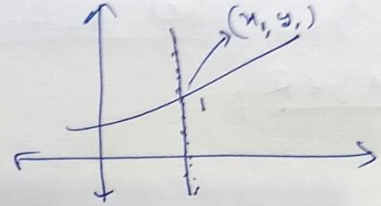
input → output ↔ value

Real Function:

those functions whose domain and range both are set of real numbers or subset of real numbers.

Graphical Definition of Function

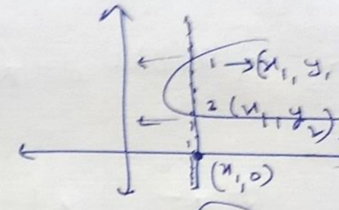
If vertical line does not cut the graph of relation 'f' for more than one point then 'f' is known as function.



f_1

✓

Fn.



f_2

X

not a Fn.

Set Builder Form:

interval

$f_1 = \{(x, y) : y = x^2,\ x, y \in [0, \infty)\}$ → Yes/no

$f_2 = \{(x, y) : x = y^2,\ x, y \in [0, \infty)\}$ → Yes/No

$f_1 = \{(0,0),$

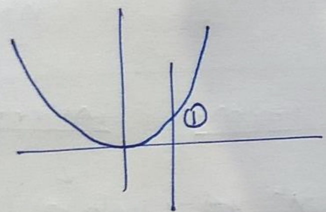
Set Builder

$f_1 = \{(x, y) : y = x^2,\ x, y \in R\}$

$f_2 = \{(x, y) : x = y^2,\ x, y \in R\}$

$y = x^2$

x	1	2	-1	-2
y	1	4	1	4

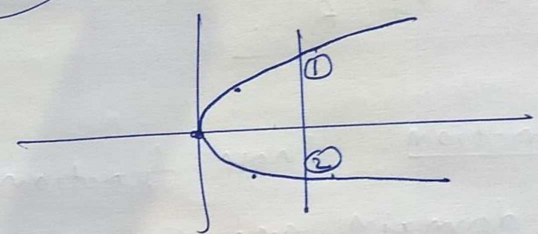


Fn ✓

$x = y^2$

$x = (-1)^2$
$= 1$

x	0	1	1	4	4
y	0	1	-1	-2	2



Not a Fn ✗

$y = x^2$ $\{(0,0), (1,1), (2,4), (-1,1), (-2,4)$ ---- $\}$

(x, y)
$\uparrow\ \ \uparrow$
$-1\ \ 1$

$x = y^2$

(x, y)

$f_2 = \{(0,0), (1,1), (1,-1), \ldots$

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Before Exercise 2.3

Inequality   Domain   Range

Most important video lecture

Inequalities ($>, <, \geq, \leq$)

$5>3$ ✓, $-2<-1$ ✓, $0 \leq 7$ ✓

$x>3$ ✓

$7y^2-5<100$ ✓

(I) let $a>b$

$\Rightarrow -a<-b$

⊖ multiply ⇓ sign of inequality change

E.g. $7<10$

$-7>-10$

(II) let $a>b$

$\Rightarrow \boxed{Ka>Kb}$

+K multiply ⇓ Sign of inequ. does not change.

(III) Reciprocal

$x<a<b$

Both limits required (same sign)

$\oplus x < a < \oplus b$	$\ominus c < d < \ominus e$	$\ominus c < a < \oplus b$
$\Rightarrow \frac{1}{x} > \frac{1}{a} > \frac{1}{b}$	$\Rightarrow \frac{1}{c} > \frac{1}{d} > \frac{1}{e}$	⇓ Please do not take reciprocal.

$-$ | $+$ on number line at 0

$\frac{1}{0} = \infty$ = Not defined

For e.g.

Solve. (I) $3x-7<2$

(II) $2<3-4x\le 10$

(III) $\frac{1}{2}<\frac{1}{2-3x}$

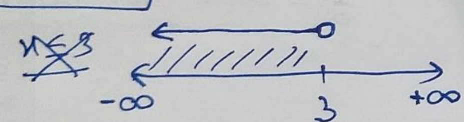
(I) $3x-7<2$

$\Rightarrow 3x<9$

$\Rightarrow \frac{3x}{3}<\frac{9}{3}$ $(\div 3)$ (+)ve No sign change

$<\Rightarrow<$

$\Rightarrow \boxed{x<3}$



$x\in(-\infty,3)$

small bracket   small bracket   (Exclude)

(II) $2<\boxed{3-4x}\le 10$

$\Rightarrow 2-3<-4x\le 10-3$

$\Rightarrow -1<\boxed{-4x}\le 7$

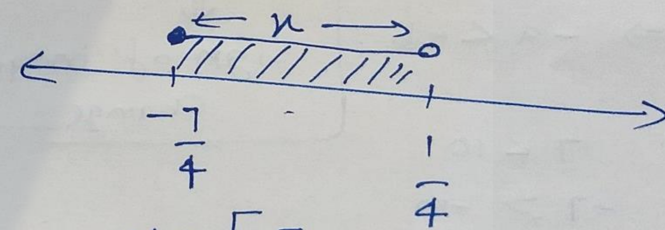
$\Rightarrow \frac{-1}{-4}>\frac{-4x}{-4}\ge\frac{7}{-4}$

($-4\Rightarrow$ (−)ve Sign change Inequality का)

$\Rightarrow \frac{1}{4}>x\ge-\frac{7}{4}$

$2>1$
$1<2$

$\Rightarrow -\frac{7}{4}\le x<\frac{1}{4}$



$x\in\left[-\frac{7}{4},\frac{1}{4}\right)$

Quadratic Inequality : (Higher Degree Inequality)

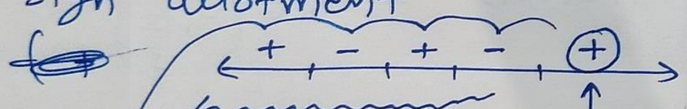
Steps. :→

(I) Make leading coefficient positive (with other side zero)

(II) Factorize

(III) Make number line & mark roots on it

(IV) Sign allotment



[then Alternate sign]

[on Right most part always $\oplus$ve]

(V) Chose either $\oplus$ve part or $\ominus$ve Part according to inequality

e.g. $-x^2 + 5x \le 6$

$\Rightarrow -(-x^2 + 5x) \ge -6$ (− multiply)

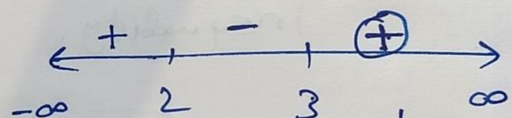
$\Rightarrow x^2 - 5x \ge -6$

$\Rightarrow x^2 - 5x + 6 \ge 0$ (I step complete)

$\Rightarrow x^2 - 2x - 3x + 6 \ge 0$

$\Rightarrow x(x-2) - 3(x-2) \ge 0$

$\Rightarrow \boxed{(x-2)(x-3) \ge 0}$ (II step complete)



$x = 2, 3$ roots.

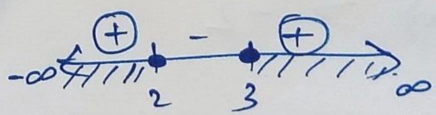
For e.g. $x = 10$

$(10-2)(10-3) \ge 0$

$(8 \times 7) = 56 \ge 0$

$(x-2)(x-3) \ge 0$

positive $\oplus$ ⟶ chose $\oplus$ Part.



Solution $= x \in (-\infty, 2] \cup [3, \infty)$

Next example.

Solve. $\boxed{100 - x^2 > 0}$

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$\boxed{100 > x^2}$

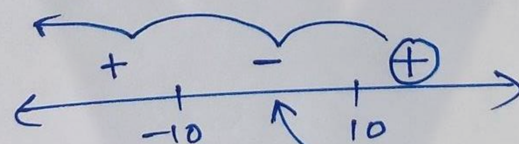
$100 - x^2 > 0$

⊖ multiply. - - -

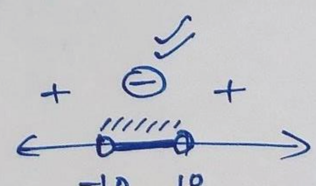
$\Rightarrow \quad x^2 - 100 < 0$ $\qquad a^2 - b^2 = (a+b)\cdot(a-b)$

$\Rightarrow \quad \boxed{(x+10)(x-10) < 0}$

$x = -10, 10$



$\underbrace{(x+10)(x-10)}_{\ominus\text{ve Number}} < 0$



$\boxed{x \in (-10, 10)}$

Solution

Domain

Inequality >>> Domain >>> Range

Domain = set of all possible inputs.

Range = set of all possible outputs.

e.g. $\boxed{f(x) = 3x - 7}$ $f(x) = y$

$f(x)$ ↑ value of x ↑ input ↑ Domain(x)

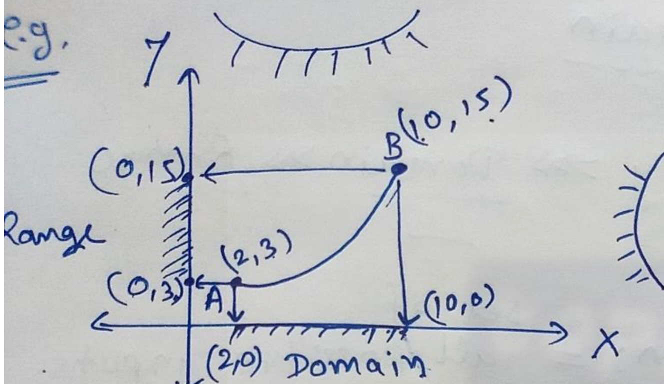
$3x - 7$ → output (y)

$$y = \overbrace{f(x)}^{\text{Range(output}}$$

y → Output (Range) → Y-axis

x ↑ input (Domain) → X-axis

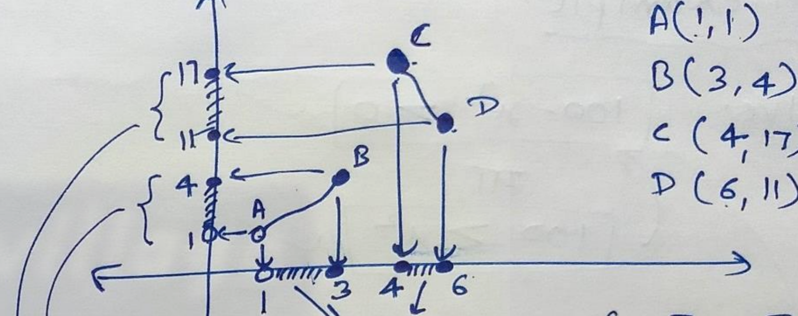
e.g.



Domain = $x \in [2, 10]$

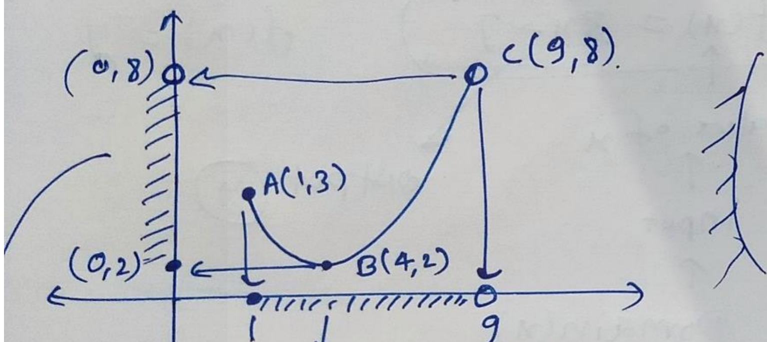
Range = $y \in [3, 15]$

A(1,1)
B(3,4)
C(4,17)
D(6,11)



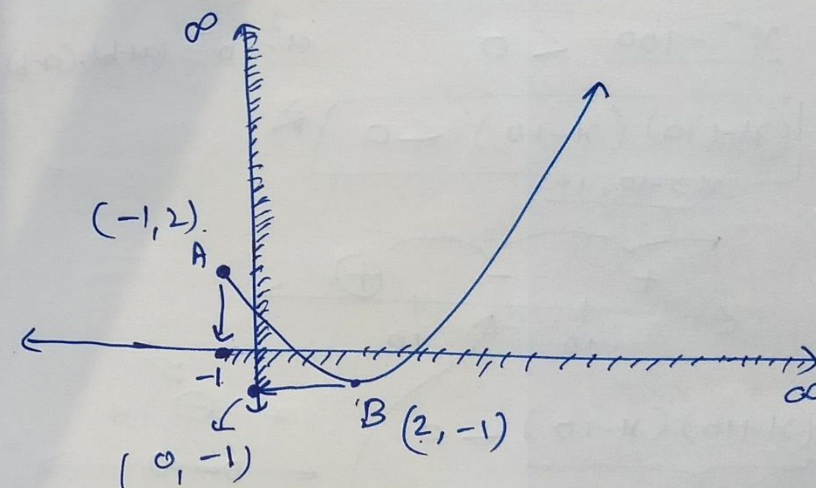
Domain = $(1, 3] \cup [4, 6]$

Range = $(1, 4] \cup [11, 17]$



Domain = $[1, 9)$

Range = $[2, 8)$



Domain = $[-1, \infty)$

Range = $[-1, \infty)$

Ways to find Domain :→

- $\sqrt{f(x)}$ ⇓ $\boxed{f(x) \geq 0}$ → Solve, $x \in$ - -
- $\dfrac{f(x)}{g(x)}$ ⇓ $\boxed{g(x) \neq 0}$ → $x \neq$ (- - -)

$\dfrac{f(x)}{0} = \infty$ ↓ Not Defined

× $\sqrt{\ominus}$ = imaginary No. (Not real)

$\sqrt{-1} = i =$ iota

$\sqrt{-7} =$

$\sqrt{0}$ ✓ Real

$\sqrt{+}$ ✓ Real

---

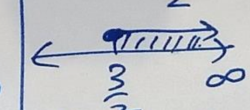
for example.

① $f(x) = \sqrt{2x-3}$

For Domain $2x - 3 \geq 0$

$\Rightarrow 2x \geq 3$

$x \geq \dfrac{3}{2}$



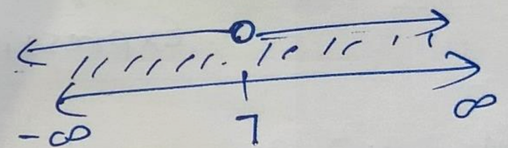
$[3/2, \infty)$

② $f(x) = \dfrac{2x+3}{x-7}$

For Domain $x - 7 \neq 0$

$\boxed{x \neq 7}$

x can be anything but not 7.



$x \in (-\infty, 7) \cup (7, \infty)$ ✓

या

$x \in (-\infty, \infty) - \{7\}$ ✓

या

$x \in R - \{7\}$ ✓

Range: $\rightarrow$ output ✓

$\downarrow$

Value of y ✓

$\downarrow$

value of $f(x)$ ✓

method $\approx$ Start with Known Function or Expression

$y = f(x) = x^2 - 3$, $x \in R$

Domain & Range.

$\downarrow$

$x \in R$

Domain $= R$ ✓

Range $y = f(x) = x^2 - 3$

$x \in R$

Known $= x^2 =$ equal to zero or positive.

But $x^2 \neq$ negative.

Perfect square $= (ax+b)^2 \geq 0$

$$\boxed{y = (x^2 - 3) \neq f(x)}$$

$x^2 \geq 0$ (Always)

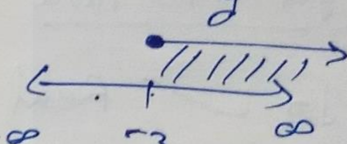
$\Rightarrow x^2 - 3 \geq 0 - 3$

$\Rightarrow (x^2 - 3) \geq -3$

$f(x) \geq -3$

या

$y \geq -3$



~~$y \in$~~ $y \in [3, \infty)$

Range $= [-3, \infty)$

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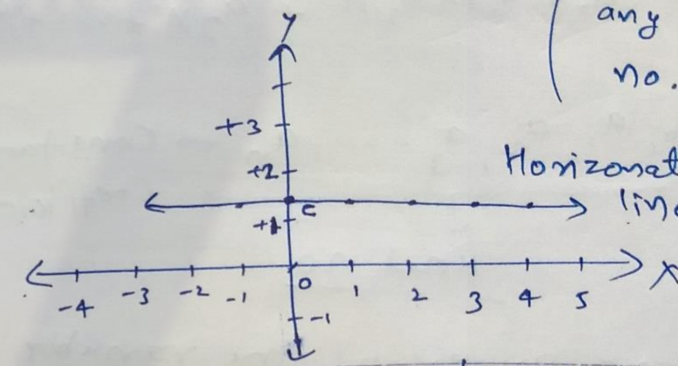
(ii) Constant Function:

$f : R \to R$ defined by $\boxed{f(x) = y = c}$

(Constant any Real no.)



x	0	1	2	3	4	-1
y	c	c	c	c	c	c

Constant output

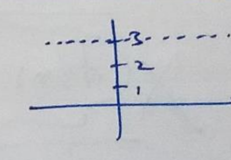
Domain $= R$.

Range $= \{c\}$ = Singleton set.

e.g. $y = f(x) = 3$

Domain $= R$

Range $= \{3\}$



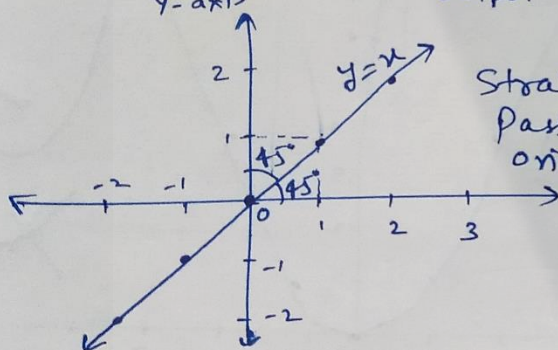
Some Functions and their Graphs

Name | Expression | Graph | ☆ Domain Range

(i) Identity Function:

$f : R \to R$ defined by $y = f(x) = x$

output $=$ input



Straight line Passing through origin (0,0)

x	0	1	2	-1	-2
y	0	1	2	-1	-2

Domain $= R$

Range $= R$

Codomain $= R$

(iii) Polynomial Functions

$f: R \to R$ defined by

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n$$

$\underbrace{a_0, a_1, a_2, a_3 \dots a_n}_{\text{Real no.}} \to$ Constant Coefficients.

n = power of the variable 'x'

= whole no. $= \{0, 1, 2, 3 \dots\}$

e.g.

$f(x) = 2x + 3$

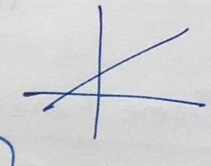
$f(x) = x$

$f(x) = 3x^3 - 5x^2 + 7x - 8$

$\times$ $f(x) = x^{2/3} - x^{100} + x^2 + 1$

$2/3 \notin W$

Note: Linear Functions



$y = \boxed{f(x) = ax + b}$, $a \neq 0$

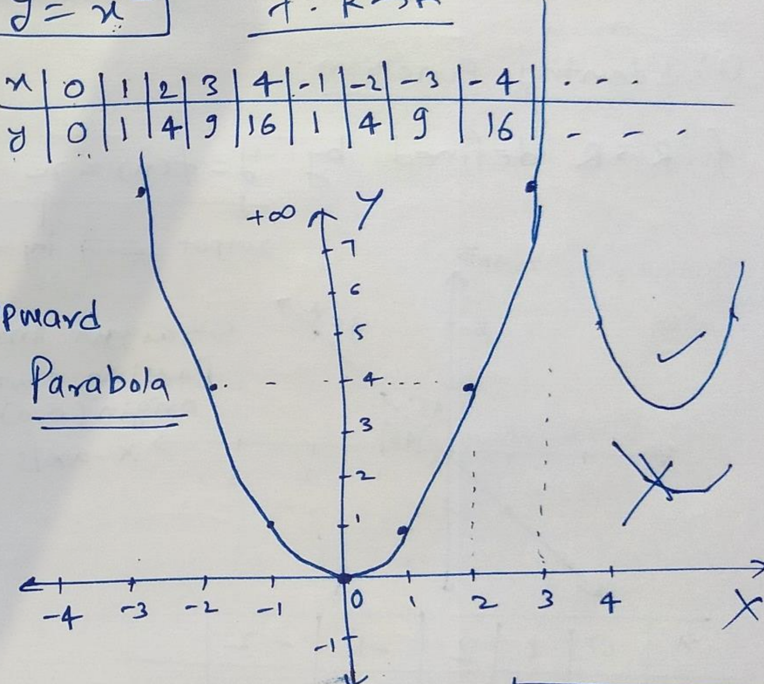
Straight line

e.g.

① $\boxed{y = x^2}$ $f: R \to R$

x	0	1	2	3	4	-1	-2	-3	-4	...
y	0	1	4	9	16	1	4	9	16	...

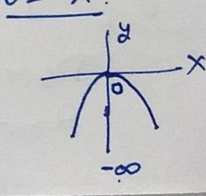
Upward Parabola



Domain = R

Range = $[0, \infty)$

$y = -x^2$



e.g. ② $y = f(x) = x^3$

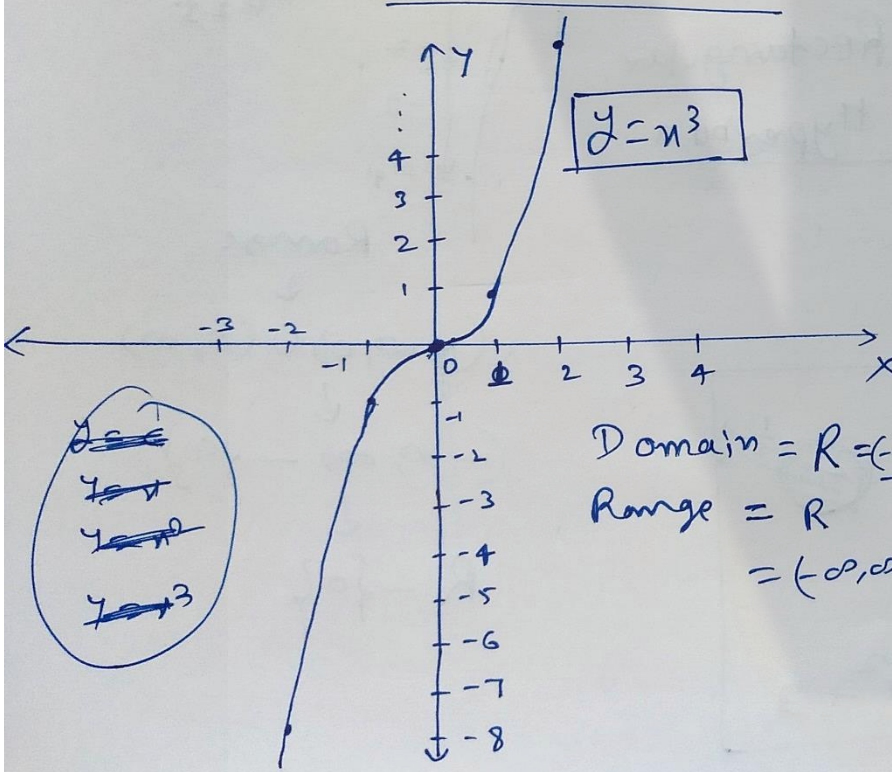
$f : R \to R$

input

$\boxed{y = x^3}$

x	0	1	2	3	−1	−2	−3	....
y	0	1	8	27	−1	−8	−27	

$(-1)^3 = -1 \cdot -1 \cdot -1 = -1$



Domain $= R = (-\infty, \infty)$

Range $= R$
$= (-\infty, \infty)$

(iv) Rational Functions: $= \dfrac{f(x)}{g(x)}$; $g(x) \neq 0$

$f(x)$, $g(x) \to$ Polynomial F_n.

e.g. $f(x) = \dfrac{2x+3}{3-x}$ Domain $\Rightarrow x \in R - \{3\}$

$3 - x \neq 0$

$x \neq 3$

$\dfrac{x}{0} = \infty$

e.g. $\boxed{y = \dfrac{1}{x}} = f(x)$ $f : R - \{0\} \to R - \{0\}$

$x \neq 0$

Domain $= R - \{0\}$

$y = \frac{1}{x} = f(x)$ $\quad f: R - \{0\} \longrightarrow R - \{0\}$

x	0.25	0.5	1	2	−0.25	−0.5	−1	−2
y	4	2	1	0.5	−4	−2	−1	−0.5

$y = \frac{1}{x} = \frac{1}{0.25} = \frac{100}{25} = 4$

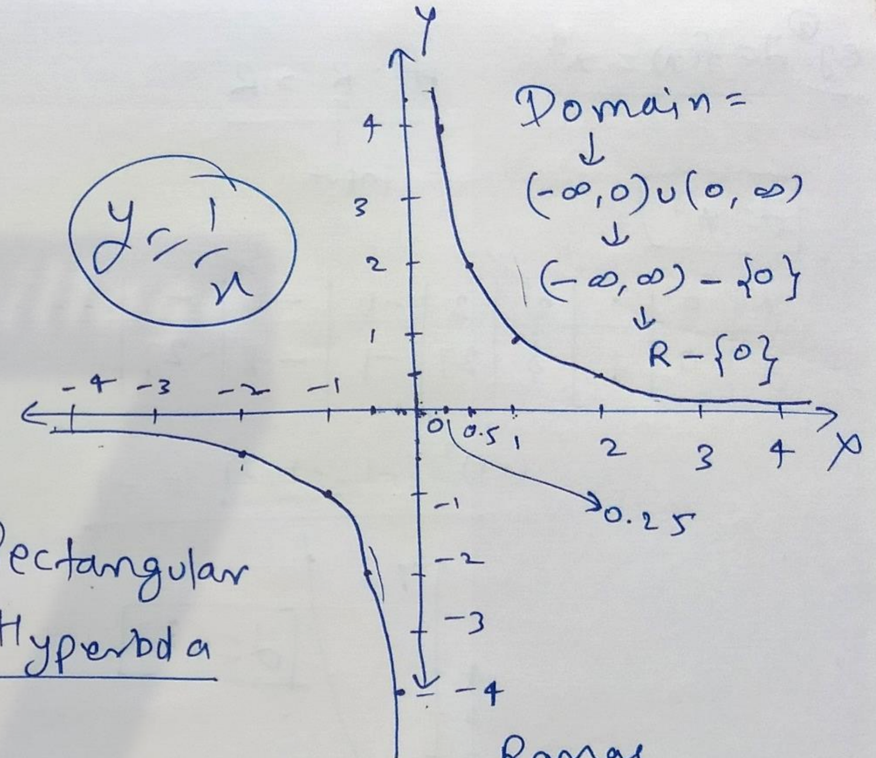
$y = \frac{1}{x} = \frac{1}{0.5} = \frac{10}{5} = 2$

$y = \frac{1}{1} = 1$

$y = \frac{1}{2} = 0.5$

$y = \frac{1}{x}$

Rectangular Hyperbola



Domain =

$\downarrow$

$(-\infty, 0) \cup (0, \infty)$

$\downarrow$

$(-\infty, \infty) - \{0\}$

$\downarrow$

$R - \{0\}$

Range

$\downarrow$

$(-\infty, 0) \cup (0, \infty)$

$\downarrow$

$(-\infty, \infty) - \{0\}$

$\downarrow$

$R - \{0\}$

Note: Irrational Fn^n.

$2, 3 \quad \frac{1}{2}, \frac{1}{3}$

$f(x) = \sqrt{x} \longrightarrow x \geq 0$

$f(x) = x^{2/3}$

$f(x) = \sqrt{(7x - 3)} + x^2 + 3$

Some functions and their graphs:

Name	Expression	Graph	Domain Range

$$y = f(x) = |x| = \begin{cases} x, & x \geq 0 \quad \text{Positive \& zero} \\ -x, & x < 0 \quad \text{Negative} \end{cases}$$

+ (Positive) = ⊕

− (Negative) = ⊕

(V) The Modulus Function
(Absolute Value Functions)

$f : R \to R$ ~~definde~~ defined by $f(x) = |x|$

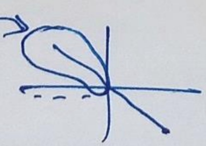
e.g. $f(2) = |2| = 2$

$f(-2) = |-2| = 2 = -(-2)$

$f(0) = |0| = 0$

$f(-3.8) = |-3.8| = 3.8$

$y = -x$



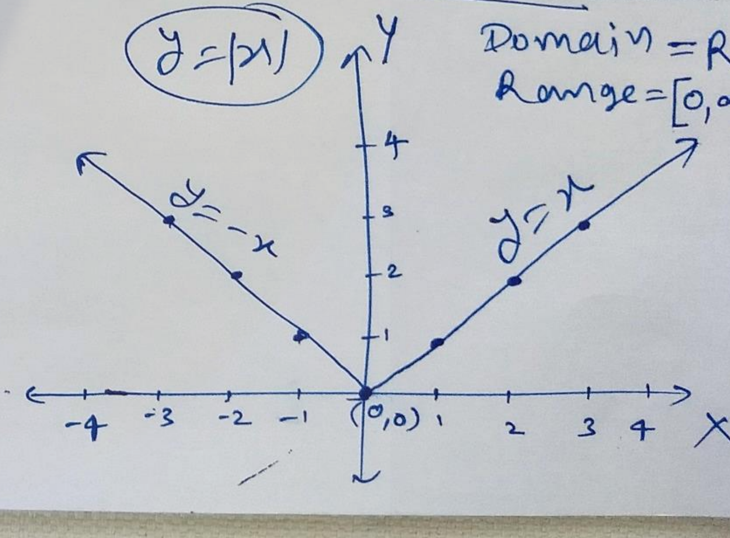
$x = -4$

$f(-4) = |-4| = \{ -(-4), \; -4 < 0$

$= 4 = 4$

$y = |x|$

x	y
0	0
1	1
2	2
−1	1
−2	2
−3	3
3	3



Domain = R

Range = $[0, \infty)$

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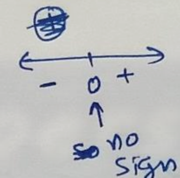
$-2 = (-1) \times 2$

(VI) Signum Function:



$f : R \to R$ defined by

$$y = f(x) = \operatorname{sgn}(x) = \begin{cases} +1 & , \text{ if } x > 0 \text{ (positive)} \\ 0 & , \text{ if } x = 0 \text{ (zero)} \\ -1 & , \text{ if } x < 0 \text{ (Negative)} \end{cases}$$

e.g. $f(2) = +1 = \operatorname{sgn}(2) \quad (\because 2 > 0)$

$f(-3) = -1 = \operatorname{sgn}(-3) \quad (\because -3 < 0)$

$\operatorname{sgn}(100) = +1$

$\operatorname{sgn}(-20) = -1$

$\operatorname{sgn}(\pi) = +1$

$\operatorname{sgn}(0) = 0$

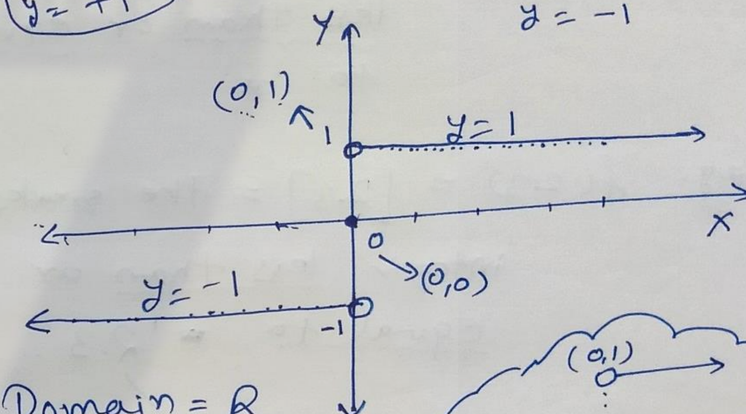
$\operatorname{sgn}(-\pi) = -1$

Graph of $\operatorname{sgn}(x)$:

x	0	1	2	3	4	...	-1	-2	-3	-4	...
$\operatorname{sgn}(x) = y$	0	1	1	1	1		-1	-1	-1	-1	

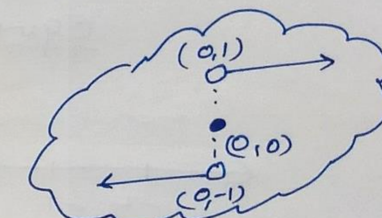
$x = 0.001$
$y = +1$

$x = -0.0001$
$y = -1$



Domain = R

Range = $\{1, 0, -1\}$



$$y = \operatorname{sgn}(x) = f(x) = \begin{cases} 1 & , x > 0 \\ 0 & , x = 0 \\ -1 & , x < 0 \end{cases} \qquad x \neq 0$$

$$= \begin{cases} 0 & , x = 0 \\ \dfrac{|x|}{x} & , x \neq 0 \end{cases}$$

$x > 0$

$x = 3$

$\frac{|3|}{3} = \frac{3}{3} = 1$

$x < 0$

$x = -10$

$\frac{|-10|}{-10} = \frac{10}{-10} = -1$

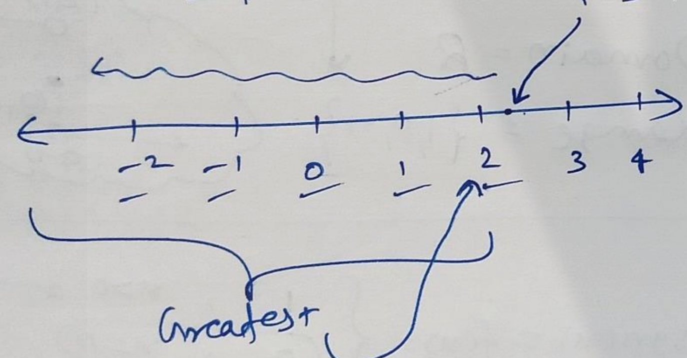
$\frac{|x|}{x} = \frac{x}{|x|}$ ☆

(VII) Greatest integer Function (GIF)

$f: R \to R$ defined by

$y = f(x) = [x]$ = the greatest integer less than or equal to 'x'

e.g. $f(2.3) = [2.3]$ = the greatest integer less than or equal to '2.3'.

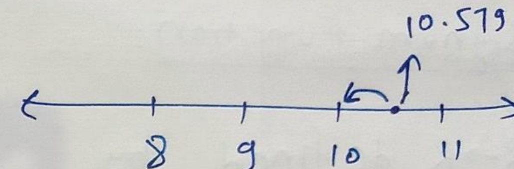


$f(2.3) = [2.3] = 2$ ✓

$[2.4] = 2$

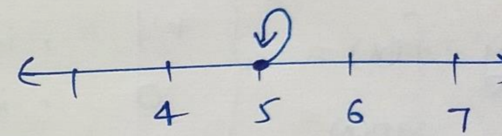
$[2.9] = 2$

e.g. $[10.579] = 10$

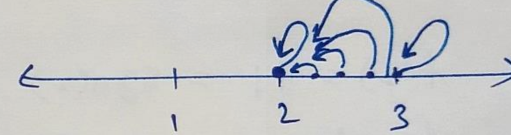


e.g. $[5] = 5$

integer

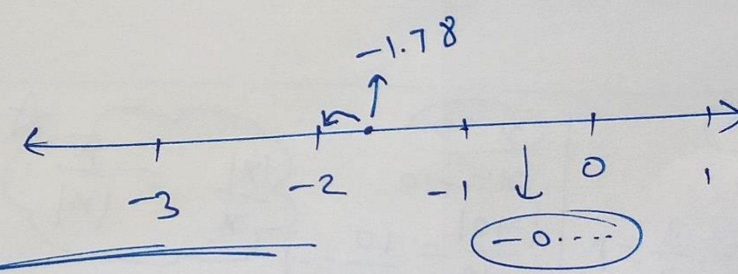


e.g.



$[2.99999] = 2$

e.g. $[-1.78] = -2$



$[0] = 0$
$[0.1] = 0$
$[0.2] = 0$
$\vdots$
$[0.9] = 0$

$[1] = 1$
$[1.1] = 1$
$[1.2] = 1$
$\vdots$
$[1.999] = 1$

$[2] = 2$
$[2.1] = 2$
$[2.2] = 2$
$\vdots$
$[2.99] = 2$

$[3] = 3$
$[3.001] = 3$
$\vdots$
$[3.99] = 3$

$[4] = 4$
$[4.01] = 4$

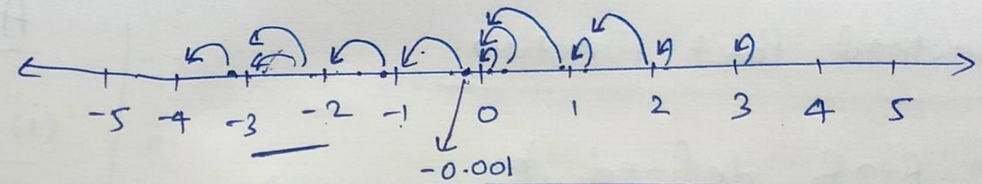
$[-0.001] = -1$
$[-0.2] = -1$
$[-0.9] = -1$
$[-1] = -1$

$[-1.1] = -2$
$[-1.5] = -2$
$[-1.9] = -2$
$[-2] = -2$

$[-2.001] = -3$
$[-2.2] = -3$
$[-2.3] = -3$
$[-2.99] = -3$
$[-3] = -3$

$[-3.1] = -4$

$[\pi] = 3$

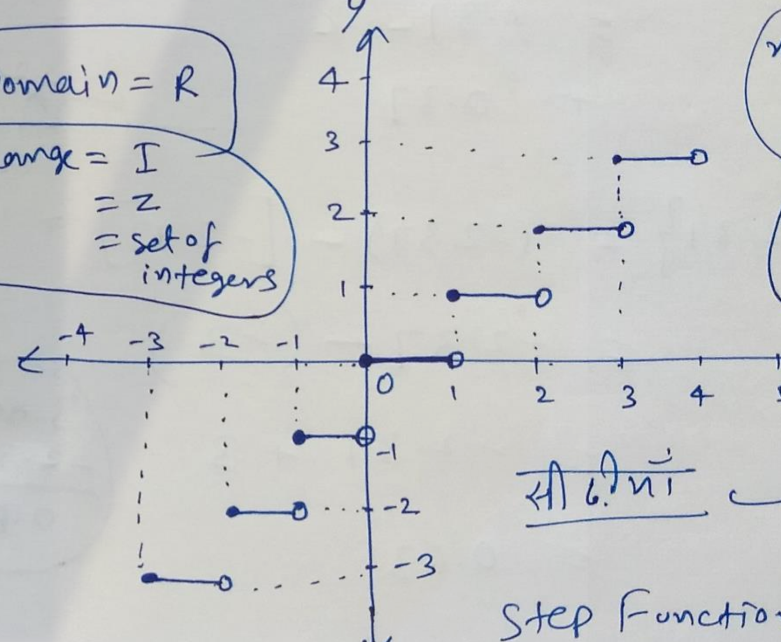


$[I] = I$
↑
integer

$$y = f(x) = [x] = \begin{cases} -3, & x \in [-3, -2) \\ -2, & x \in [-2, -1) \\ -1, & x \in [-1, 0) \\ 0, & x \in [0, 1) \rightarrow 0 \le x < 1 \\ 1, & x \in [1, 2) \rightarrow 1 \le x < 2 \\ 2, & x \in [2, 3) \rightarrow 2 \le x < 3 \\ y = 3, & x \in [3, 4) \end{cases}$$

Domain = R

Range = I
= Z
= set of integers



$x \in [0, 1)$
↓
$y = 0$

$x \in [1, 2)$
↓
$y = 1$

$Y = 0$

सीढ़ीनां

Step Function

Fractional Part Function.

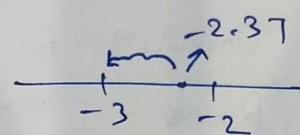
$f: R \to R$ defined by

$$y = f(x) = \boxed{\{x\} = x - \underbrace{[x]}_{\text{GIF}}}$$

$$\{2.37\} = 2.37 - \underbrace{[2.37]}_{\text{GIF}}$$

$$= 2.37 - 2$$

$$= 0.37$$



$$\{-2.37\} = (-2.37) - [-2.37]$$

$$= -2.37 - (-3)$$

$$= -2.37 + 3$$

$$= 0.63$$

$$\begin{array}{r} 3.00 \\ -2.37 \\ \hline 0.63 \end{array}$$

Algebra of Real Functions.

(i) Addition

$$(f+g)(x) = f(x) + g(x)$$

(ii) Subtraction =

$$f(x) - g(x) = (f-g)(x)$$

(iii) multiplication

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

(iv) Division

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad g(x) \neq 0$$

(v) Scalar muliplication

$$(\alpha f)(x) = \alpha . f(x)$$

e.g. $f(x) = 2x+3 \quad x \in R$

$g(x) = x-7 \quad x \in R$

① $(f+g)(x) = f(x)+g(x) = 2x+3+x-7 = 3x-4 \quad , x \in R$

② $(f-g)(x) = f(x)-g(x) = (2x+3)-(x-7) = x+10 \quad , x \in R$

③ $(f \cdot g)(x) = (2x+3)(x-7) = 2x^2-11x-21 \quad , x \in R$

④ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)_{\neq 0}} = \frac{2x+3}{x-7 \neq 0}$ Domain $= x \in R - \{7\}$

⑤ $(7f)(x) = 7 \cdot f(x) = 7(2x+3) = 14x+21 \quad , x \in R$

$(8g)(x) = 8 \cdot g(x) = 8(x-7) = 8x-56 \quad , x \in R$

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Exercise $\boxed{2.3}$ {all questions are related to functions}

① (i)

$\{(2,1), (5,1), (8,1), (11,1), (14,1), (17,1)\}$ Fn^n. ✓

Fn^n → first set should be Domain.

\+

A → B (No two ordered pairs have same first element) ✓

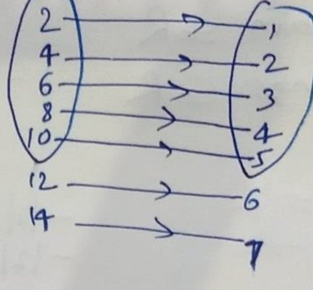
Domain = {2, 5, 8, 11, 14, 17}

Range = {1}

(ii)

$\{(2,1), (4,2), (6,3), (8,4), (10,5), (12,6), (14,7)\}$

? → ( )



Fn^n. ✓

Domain = {2, 4, 8, 6, 10, 12, 14}

Range = {1, 2, 3, 4, 5, 6, 7} ✓

(iii) $\{(1,3), (1,5), (2,5)\}$

(1,3), (1,5) → Same first element

∴ Not a function

Exercise (2.3)

(2) Domain and Range of the following real function: →

input → Domain $\subseteq R$

output → Range $\subseteq R$

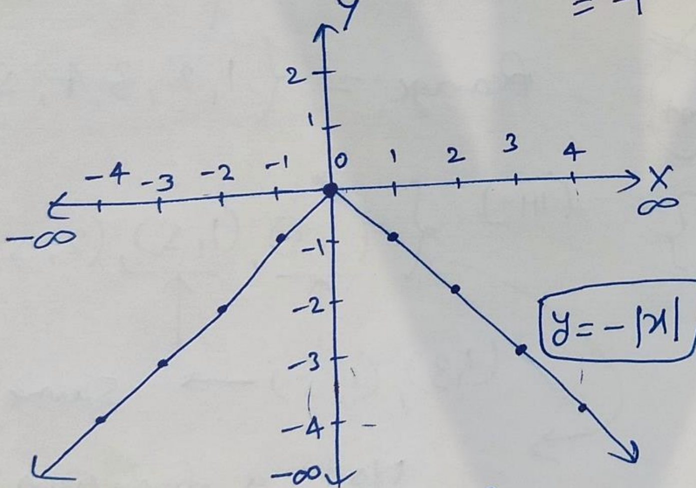
(i) $f(x) = \underbrace{-|x|} = y$

$-|1| = -1$

$-|2| = -2$

$-|-1| = -(+1)$
$= -1$

x	y
0	0
1	−1
2	−2
3	−3
4	−4
−1	−1
−2	−2
−3	−3
−4	−4
−5	−5



Domain $= (-\infty, \infty) = R$

(y) Range $= (-\infty, 0] \rightarrow \boxed{-\infty < y \leq 0}$

Second method

$f(x) = -|x|$

Domain → set of all possible inputs (which can be put in the function)

x

\# $\sqrt{g(x)} \rightarrow g(x) \geq 0$

\# $\dfrac{g(x)}{h(x)} \rightarrow h(x) \neq 0$

Domain of $f(x) = -|x|$ is R.

Range: $y = f(x) = -|x|$

(Start with known expression)

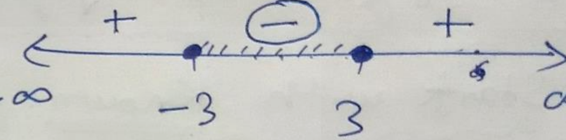
$|x| \geq 0 \rightarrow 0 \geq -|x|$

$-|x| \leq 0$

$\Rightarrow y \leq 0 \Rightarrow -\infty < y \leq 0$

$y \in (-\infty, 0]$

② (ii) $\boxed{f(x) = \sqrt{9-x^2}}$



Domain

(For Real Funⁿ.) ⇒ $9 - x^2 \geq 0.$

Coeff. of x^2 ⇓ (+)

$x = 10$ → $\sqrt{9-100}$

$\sqrt{-91}$

↓

imaginary

(not real)

~~multipp~~ multiply ⊖ ↓ (sign of inequality changes)

⇒ $x^2 - 9 \leq 0$

⇒ (factorisation)

⇒ $x^2 - 3^2 \leq 0$

$a^2 - b^2 = (a+b)(a-b)$

⇒ $(x-3)(x+3) \leq 0$

Number line ↓ mark (roots)

roots
$x = 3$
$x = -3$

✓ (sign allotment) → right most part = (+)

then alternate sign while moving left.

✓ Solution → part.
(According to sign of inequality)

$(x-3)(x+3) \leq 0$

⊖ve ← (less than or equal to)

किनारे (corners) include.

$x \in [-3, 3]$ — interval

$-3 \leq x \leq 3$ — Domain

Range of $y = f(x) = \sqrt{9-x^2}$

{ Start with Known function }

By Domain, we Know that

$x \in [-3, 3]$

$\Rightarrow -3 \le x \le 3$

$\Rightarrow$ Square

$\cancel{9 \le x^2 \le 9.}$

$x^2 = 9$

$x = 2$

$x^2 = 4$

$x^2 = x \cdot x = \oplus$
$\quad - \; -$
$\quad + \; + \quad \oplus$

$-3 \le x \le 3$

$0 \le x^2 \le 9$

$x^2 = 0$ $\quad x^2 = 1$ $\quad x^2 = 4$ $\quad x^2 = 9$

Universal truth for real x $\longrightarrow x^2 \ge 0$

$0 \le x^2 \le 9$

multiply with $\ominus$

$\Rightarrow 0 \ge -x^2 \ge -9$

$\Rightarrow -9 \le -x^2 \le 0$

$\Rightarrow 9-9 \le 9-x^2 \le 9+0$

$\Rightarrow 0 \le 9-x^2 \le 9$

Square root $\quad \pm$

$0 \le \sqrt{9-x^2} \le 3$

$\Rightarrow 0 \le y \le 3$

Range $= [0, 3]$

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Exercise 2.3 × Relations & Functions

③ $f(x) = 2x - 3$.

(i) $f(0) = 2(0) - 3 = 0 - 3 = -3$

(ii) $f(7) = 2(7) - 3 = 14 - 3 = 11$

(iii) $f(-3) = 2(-3) - 3 = -6 - 3 = -9$

④ input input

$t(c) = \frac{9c}{5} + 32$

output = F

't' is a function to change celsius into Fahrenheit.

(i) $t(0) = \frac{9(0)}{5} + 32 = 32$ Fahrenheit

0° celsius

(ii) $t(28)$

$t(c) = \frac{9c}{5} + 32$

$t(28) = \frac{9 \times 28}{5} + 32$

$= \frac{252}{5} + \frac{32}{1} = \frac{252 + 160}{5}$

$= \frac{412}{5}$ Fah.

(iii) $t(-10) = \frac{9(-10)}{5} + 32$

$= 9(-2) + 32 = -18 + 32$

$= 14$

(iv) The value of c, when $t(c) = 212$

$\frac{9c}{5} + 32 = 212$

$\Rightarrow \frac{\cancel{9}c}{5} = \cancel{180}\ 20$

$\Rightarrow c = 20 \times 5 = 100$

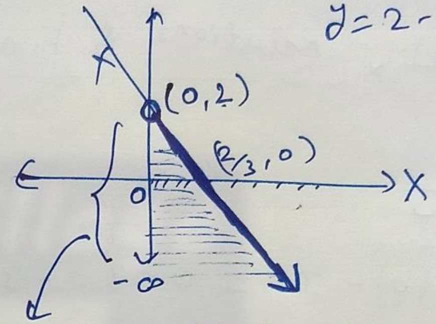
(5) Find the range:

(i) $f(x) = 2 - 3x$, $x \in R$, $x > 0$

(Start with Known function)

$y = \boxed{f(x) = 2 - 3x}$

$y = 2 - 3x$



Range $(-\infty, 2)$

Right side graph

we Know that $x > 0$ (Given)

multiply -3 = ove

$\downarrow$

sign change

Inequality

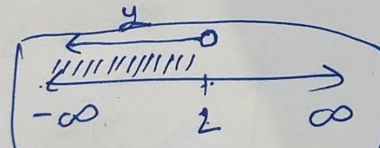
$\Rightarrow$ $-3x < 0$

add 2

$\Rightarrow$ $2 - 3x < 2 + 0$

$\Rightarrow$ $(2 - 3x) < 2$

$f(x) < 2$



$-\infty < y < 2$

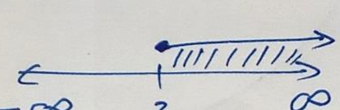
$y \in (-\infty, 2)$

(ii) $f(x) = x^2 + 2$, x is any real no.

$x \in R$

we know that

$x^2 \geq 0$



$\Rightarrow$ $(x^2 + 2) \geq 0 + 2$

$y = f(x) \geq 2$

Range $= [2, \infty)$

(iii) $y = f(x) = x$, $x \in R$ (Identity Fn^n.)

$\Rightarrow$ $x \in R$

$\Rightarrow$ $y \in R$

y = output = Range.

$\rightarrow$ Range = R

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Miscellaneous Exercise 2

Chapter - 2

Relations and Functions

1

$$f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$$

$$g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$$

$f(3) = 3^2, \ (x^2)$

$= 9.$

$f(3) = 3 \times 3, \ (3x)$

$= 9$

$(3, 9)$

3 has unique image

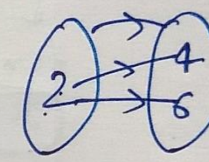
$g(2) = 2^2$ (according to x^2)

$= 4$

$g(2) = 3(2)$ (according to $3x$)

$= 6$

$(2, 4), \ (2, 6)$



2 doesn't have unique image.

$\therefore$ f is a function but g is only a relation.

---

2 $f(x) = x^2$

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1)^2 - (1)^2}{1.1 - 1} = \frac{1.21 - 1}{0.1}$$

$$= \frac{0.21}{0.1} = 2.1$$

(3) Find the domain of the function

$$f(x) = \frac{x^2+2x+1}{x^2-8x+12} \;\; .= \frac{Nr.}{Dr. \neq 0}$$

Domain: $\longrightarrow \sqrt{f(x)} \longrightarrow f(x) \geq 0$

$\dfrac{f(x)}{g(x)} \longrightarrow g(x) \neq 0$

For Domain
Denom.. $= x^2 - 8x + 12 \neq 0$

$\Rightarrow x^2 - 2x - 6x + 12 \neq 0$

$\Rightarrow x(x-2) - 6(x-2) \neq 0$

$\Rightarrow (x-2)(x-6) \neq 0$

$x \neq 2 \;, \; x \neq 6$

x can be anything but not 2 & 6.

$x \in R - \{2, 6\}$ = Domain

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Miscellaneous Exercise

Chapter (2)

(4) $f(x) = \sqrt{x-1}$

Domain Square root $\cancel{x-1<0}$

$x-1 \geq 0$

$\Rightarrow x \geq 1$

Domain = $[1, \infty)$ $x \in [1, \infty)$

Range: $y = \sqrt{x-1}$ $\sqrt{10-1}$

x	1	2	3	4	5	10
y	0	1	$\sqrt{2}$	$\sqrt{3}$	2	3

$y \in [0, \infty)$

$y = \sqrt{1-x}$

Range: Start with Known Function.

Due to Domain $x-1 \geq 0$

$\sqrt{x-1} \geq 0 \Rightarrow y \geq 0$

(5) $f(x) = |x-1|$

Domain neither $\sqrt{\ldots}$ nor $\frac{Nr}{Dr}$

$x = (-)$

$x = (+)$

$x = 0$

we can put all real numbers.

Domain = $x \in R$

Range. $y = |x-1|$ $|x| \geq 0$

x	y
0	1
1	0
2	1
3	2
-1	2
-2	3
-3	4

0, (+)ve

Range = $[0, \infty)$

we know that

$|x-1| \geq 0$

$y \geq 0$

$\Rightarrow y \in [0, \infty)$

⑥

$$f = \left\{ \left(x, \frac{x^2}{1+x^2}\right) : x \in R \right\}$$

$$f = \{ (x, y) : x \in R, \text{ relation b/w } x \,\&\, y \}$$

Ordered pair

fn^n

$$f = \left\{ (x, y) : x \in R, \ y = \frac{x^2}{1+x^2} \right\}$$

$$f(x) = y = \frac{x^2}{1+x^2}, \quad x \in R$$

I-method Range

↓

Start with known fn^n.

$$y = \frac{x^2}{1+x^2}$$

$$y = \frac{x^2+1-1}{\boxed{1+x^2}} = \frac{x^2+1}{x^2+1} - \frac{1}{1+x^2}$$

$$y = 1 - \frac{1}{1+x^2}$$

Single place of x

Start with known fn^n.

we know that $x^2 \geq 0$

$$\Rightarrow 1 + x^2 \geq 1 + 0$$

$$\Rightarrow \infty > 1 + x^2 \geq 1$$

$\frac{1}{\infty} = 0$

Reciprocal. (Sign of In. → change)

$\frac{1}{1} = 1$

$$\Rightarrow 0 < \frac{1}{1+x^2} \leq 1$$

$$\Rightarrow 0 > \frac{-1}{1+x^2} \geq -1$$

$$\Rightarrow 1 > \boxed{1 - \frac{1}{1+x^2}} \geq 1-1$$

$$\Rightarrow 1 > y \geq 0$$

$$\Rightarrow 0 \leq y < 1$$

$$y \in [0, 1)$$

II - method. Range = ? ($x \in R$)

$\boxed{\text{Represent 'x' in terms of 'y'}}$

$y = \frac{x^2}{1+x^2}$ ← Now y is represented in terms of x

$\Rightarrow y + y.x^2 = x^2$

$\Rightarrow y = -y.x^2 + x^2$

$\Rightarrow x^2.(-y+1) = y$

$\Rightarrow x^2 = \frac{y}{1-y}$

$\Rightarrow x = \pm\sqrt{\frac{y}{1-y}}$ ✓

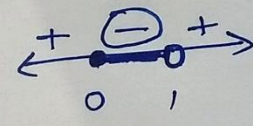
Given
$x \in R$
x is Real. ✓

$\sqrt{-}$ ✗

$\sqrt{\frac{y}{1-y}} \rightarrow$ Real ✓

$\frac{y}{1-y} \geq 0 \longrightarrow \frac{y}{-(y-1)} \geq 0$

$\Rightarrow \boxed{\frac{y}{y-1} \leq 0}$



$y \in [0, 1)$

$y = 0 \rightarrow \boxed{y = 0}$

$y - 1 = 0 \rightarrow y = 1$

Always Exclude

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Misc. Chapter 2

7. $f, g : R \to R$ | $f(x) = x+1$, $g(x) = 2x-3$

$f+g$

$\downarrow$

$(f+g)(x) = f(x) + g(x)$

$= (x+1) + (2x-3)$

$= 3x - 2$ ✓ $x \in R$

$f-g \to (f-g)(x) = f(x) - g(x)$

$= (x+1) - (2x-3)$

$= -x + 4$ ✓ $x \in R$

$\frac{f}{g} \to \frac{f(x)}{g(x)} = \frac{x+1}{2x-3}$ ✓

Dr. $\neq 0$

$2x-3 \neq 0$

$x \neq 3/2$

$x \in R - \{\frac{3}{2}\}$

8. $f = \{(1,1), (2,3), (0,-1), (-1,-3)\} \to (x, y)$

$\boxed{Z \text{ to } Z}$

$y = f(x) = ax+b$

$(1,1)$

$1 = a \times 1 + b$

$\Rightarrow \boxed{a+b=1}$ — (1)

$y = ax+b$

$(0,-1) \equiv (x, y)$

$\Rightarrow -1 = a(0) + b$

$\Rightarrow -1 = 0 + b$

$\Rightarrow \boxed{b=-1}$ — (2)

Domain $= \{1, 2, 0, -1\}$

Range $= \{1, 3, -1, -3\}$

$b = -1$ ✓

By eq^n. (1)

$a + (-1) = 1$

$\Rightarrow a = 2$ ✓

(9) R relation from N to N

$R = \{(a,b),\ a,b \in N,\ a = b^2\} \rightarrow (a,b) \in R$

(i) $(a,a) \in R$, for all $a \in N$

$(1,1) \in R$ $\quad a = 1, 2, 3, 4, \ldots$

$1 = 1^2$ ✓

$(2,2) \in R$

$2 = 2^2$ ✗

False

when $(a,b) \in R \Rightarrow a = b^2$

$(a,a) \in R \Rightarrow a = a^2$

$\Rightarrow a^2 - a = 0$

$\Rightarrow a(a-1) = 0$

$a = 0$ ✗ $\quad a = 1 \in N$

$0 \notin N$ | only one value of a,

not for all $a \in N$

False

(ii) $(a,b) \in R$, implies $(b,a) \in R$

$a = b^2 \not\rightarrow b = a^2$

Let $\left.\begin{array}{l} a = 9 \\ b = 3 \end{array}\right\}$ $9 = 3^2$

(b,a)

$(a,b) \equiv (9,3) \in R \rightarrow (3,9) \in R$???

$3 = 9^2$

$\Rightarrow 3 = 81$ ✗

False

(iii) if $(a,b) \in R$, $(b,c) \in R$ implies $(a,c) \in R$

Let

$a = (b)^2$ $\quad b = c^2 \Longrightarrow$ Prove $\boxed{a = c^2}$???

$a = (c^2)^2$

$\Rightarrow \boxed{a = c^4}$ ✓

False

Toppers Village | Misc. Ex. 2

(10)

$A = \{1, 2, 3, 4\}$

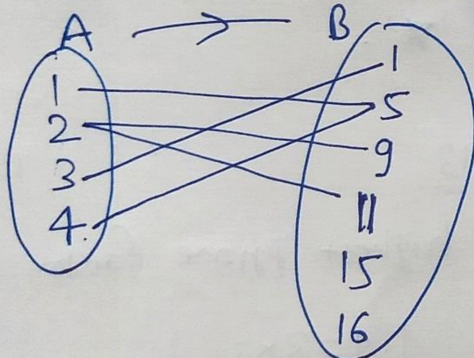
$B = \{1, 5, 9, 11, 15, 16\}$

$f = \{(1,5), \underline{\underline{(2,9)}}, (3,1), (4,5), \underline{\underline{(2,11)}}\}$

(i) f is a relation from A to B

(ii) f is a function from A to B.

(i)



True.

f is a relation from A to B.

Domain $= \{1, 2, 3, 4\} = A$

Range $= \{1, 5, 9, 11\} \subset B$

(ii)

function

(i) 1^{st} set A = Domain ✓

(ii) each element of set A

↓

must has unique image in set B.

2 has 2 images.

$\Rightarrow$ f is not a function

~~(11) f is subset of $Z \times Z$~~

~~$f = \{$ ab~~

(11) f is subset of $\mathbb{Z}\times\mathbb{Z}$

~~$f = \{(ab, a+b)\}$~~

$f = \{(ab, a+b) : a, b \in \mathbb{Z}\}$

$f = \{(x, y) : \_\_\_\_\_\_\}$

Is f function from $\mathbb{Z}$ to $\mathbb{Z}$?

$(x, y) \in f$

$10 = x = ab$

$y = a+b$

$a, b \in \mathbb{Z}$

let $a = 10$, $b = 1$

$a\times b = 10\times 1 = 10 = x$

$y = a+b = 10+1 = 11$

$(x, y) \in f$

$(10, 11) \in f \longleftrightarrow (10, 7) \in f$

let $a = 2$, $b = 5 \rightarrow x = ab = 2\times 5 = 10$

$\rightarrow y = a+b = 2+5 = 7$

$\because$ '10' has 2 images.

$\Rightarrow f$ is not a function.

---

(12) $A = \{9, 10, 11, 12, 13\}$

$f : A \to \mathbb{N}$

$f(n)$ = the highest prime factor of n.

$f(x)$ = the highest prime factor of 'x'.

input. A

$x = 9, 10, 11, 12, 13$

$10 = 2\times 5$

$f(10)$ = the highest prime factor of '10'

$= 5$

$f(9) = 3$

$f(11) = 11$

$f(12) = 3$

$f(13) = 13$

$9 = 3\times 3$

$11 = 1\times 11$

$12 = 2\times 3\times 2$

$13 = 1\times 13$

Range $= \{3, 5, 11, 13\}$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - https://www.youtube.com/toppersvillage

- **Website :** www.toppersvillage.com

- **Instagram :** https://www.instagram.com/toppersvillage/

- **Facebook Page:** www.facebook.com/ToppersVillage/

- **Facebook Group:** https://www.facebook.com/groups/ToppersVillage