



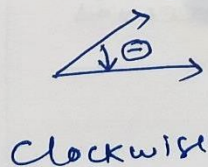
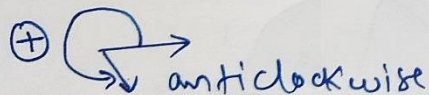
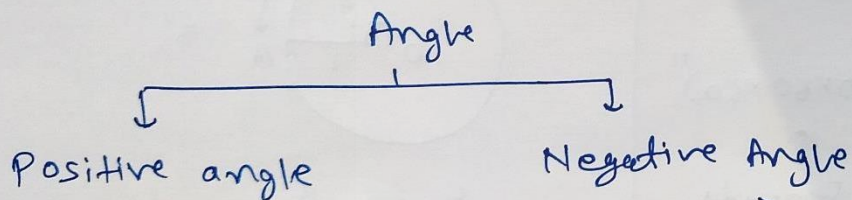
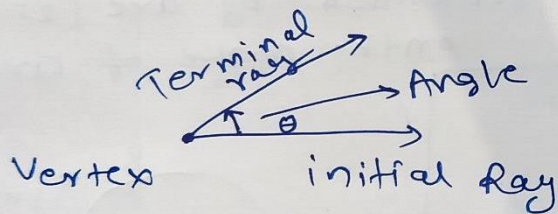
Toppers Village

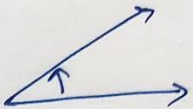
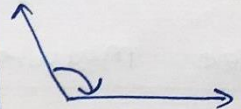
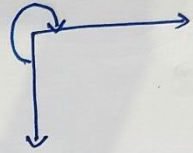
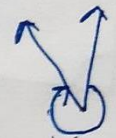


Class - 11 Maths

Chapter - 3 Trigonometric Functions

Exercise 3.1 \* Basics of Angles

Angles: measurement of rotation between two arms (rays)

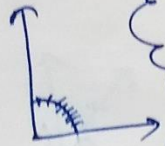


 $+ 30^\circ$	 $- 120^\circ$
 $- 270^\circ$	 $- 340^\circ$
 $+ 360^\circ$	 $- 360^\circ$

One complete revolution

## Degree measurement:

A right angle is divided into 90 parts.



each part is known as one degree =  $1^\circ$



$$1^\circ = 60 \text{ parts} = 60 \text{ minutes} = 60'$$

$$1 \text{ minute} = 60 \text{ parts} = 60 \text{ seconds} = 60''$$

$$1 \text{ right angle} = 90^\circ = (90 \times 60)' = (90 \times 60 \times 60)''$$

↑  
General language

↑  
Degree

↑  
minute

↑  
seconds

## Radian Measurement:

[Circular System]

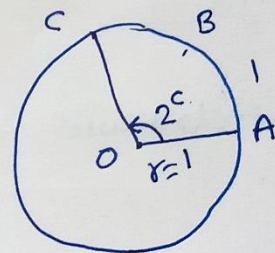
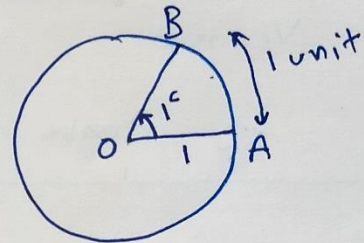
$$1 \text{ radian} = 1^\circ = 1$$

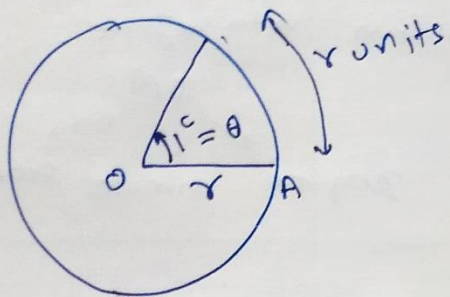
Definition of unit circle

$$r = 1 \text{ unit}$$

Definition of one radian

In a circle ( $r=1$  unit), angle subtended by arc ( $l=1$  unit) on the centre of circle =





$$\theta = \frac{l}{r}$$

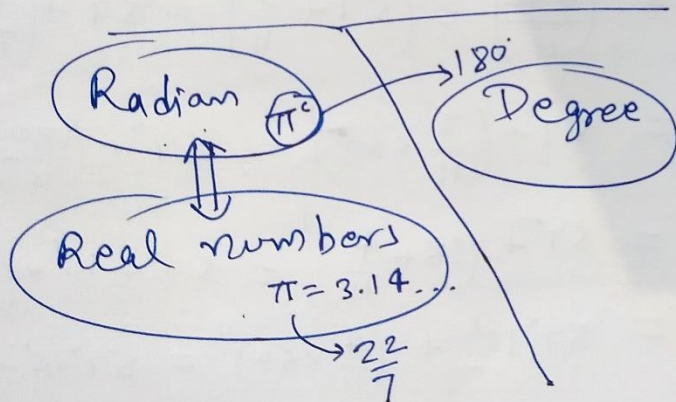
length of arc

radius of circle

Angle subtended by arc on the centre

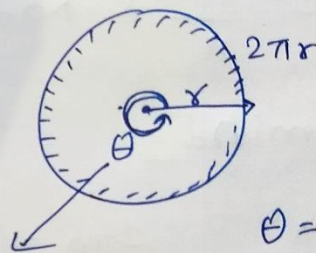
only in radian

Note :



## Relation b/w Degree & Radian

one complete revolution =  $\frac{360^\circ}{1} = \frac{2\pi^c}{1}$



$$\theta = \frac{l}{r}$$

$$\theta = \frac{2\pi r}{r} = 2\pi$$

$$\theta = 2\pi = 2\pi \text{ radian} = 2\pi^c$$

So Relation is

$$360^\circ = 2\pi^c$$

$$270^\circ = \left(\frac{3\pi}{2}\right)^c$$

$$180^\circ = \pi^c$$

$$90^\circ = \left(\frac{\pi}{2}\right)^c$$

$$45^\circ = \left(\frac{\pi}{4}\right)^c$$

$$1^\circ = \left(\frac{2\pi}{360}\right)^c$$

$$270^\circ = \left(\frac{\pi}{180}\right) \cdot 270$$

↑ Degree      ↑ Rad.



e.g.

Convert  $40^{\circ}30'$  into radian.

$$\text{radian} = \frac{\pi}{180} \times \underline{\text{Degree}}$$

$$= \frac{\pi}{180} \times (40^{\circ}30')$$

$$= \frac{\pi}{180} \times \left(40^{\circ} + \frac{30^{\circ}}{60}\right)$$

$$= \frac{\pi}{180} \times \left(40 + \frac{1}{2}\right)^{\circ}$$

$$= \frac{\pi}{180} \times \left(\frac{81}{2}\right)^{\circ}$$

$$= \frac{\pi \times 9}{20 \times 2} = \frac{9\pi}{40} \text{ radian}$$

$$\underline{40^{\circ}30' = \frac{9\pi}{40} \text{ radian}}$$

radian  $\pi$   
↓  
Degree  $180^{\circ}$

Toppers Village

Class - 11 Maths

Exercise 3.1

①

$$(i) 25^\circ = \frac{5\pi}{36}$$

$$\text{Radian} = \frac{\pi}{180} \times \text{Degree}$$

$$= \frac{\pi}{180} \times 25^\circ$$

$$= \frac{5\pi}{36}$$

$$\frac{5\pi}{36}$$

$$(ii) -(47^\circ 30')$$

$$= -(47^\circ + (\frac{30'}{60}))$$

$$= -(47 + \frac{1}{2})^\circ$$

$$= -(\frac{95}{2})^\circ$$

$$\begin{aligned} \text{Radian} &= \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \\ &= \frac{-19\pi}{72} \end{aligned}$$

$$(iii) 240^\circ$$

$$\text{Radian} = \frac{\pi}{180} \times 240 = \frac{4\pi}{3}$$

$$(iv) 520^\circ$$

$$\text{Radian} = \frac{\pi}{180} \times 520 = \frac{26\pi}{9}$$

$$(2) \text{ (i) } \left(\frac{11}{16}\right)^c$$

$$\text{Degree} = \left(\frac{180}{\pi} \times \text{Radian}\right)$$

$$\left(\pi = \frac{22}{7}\right)$$

$$= \frac{180}{\left(\frac{22}{7}\right)} \times \frac{11}{16}$$

$$= \frac{180}{22} \times 7 \times \frac{11}{16}$$

$$\begin{array}{r} 8 \overline{) 315} \quad (39) \\ \underline{32} \\ 35 \\ \underline{32} \\ 3 \end{array}$$

$$= \left(\frac{315}{8}\right)^{\circ}$$

$$= \left(39 \frac{3}{8}\right)^{\circ}$$

$$= 39^{\circ} \left(\frac{3}{8}\right)^{\circ} = 39^{\circ} + \left(\frac{3}{8} \times 60\right)^{\prime}$$

$$\Rightarrow 39^{\circ} + \left(\frac{45}{2}\right)^{\prime}$$

$$= 39^{\circ} + \left(22 \frac{1}{2}\right)^{\prime}$$

$$= 39^{\circ} + 22' + \left(\frac{1}{2}\right)^{\prime}$$

$$= 39^{\circ} + 22' + \left(\frac{1}{2} \times 60\right)^{\prime\prime}$$

$$= 39^{\circ} + 22' + 30''$$

$$= 39^{\circ} 22' 30''$$

$$\text{(ii) } (-4)^c$$

$$\pi = \frac{22}{7}$$

$$\text{Degree} = \frac{180}{\pi} \times (-4)$$

$$= \frac{180}{22} \times 7 \times (-4)$$

$$= - \frac{1260}{11} \times (2)$$

$$= - \left(\frac{2520}{11}\right)^{\circ}$$

$$\begin{aligned}
&= -\left(\frac{2520}{11}\right)^\circ \\
&= -\left(229 \frac{1}{11}\right)^\circ \\
&= -\left(229^\circ + \left(\frac{1}{11}\right)^\circ\right) \\
&= -\left(229^\circ + \left(\frac{60}{11}\right)'\right) \\
&= -\left(229^\circ + \left(5 + \frac{5}{11}\right)'\right) \\
&= -\left(229^\circ + 5' + \left(\frac{5}{11}\right)'\right) \\
&= -\left(229^\circ + 5' + \left(\frac{5}{11} \times 60\right)''\right) \\
&= -\left(229^\circ + 5' + \frac{300}{11}''\right) \\
&= -\left(229^\circ + 5' + 27''\right) \\
&= \underline{-229^\circ 5' 27''}
\end{aligned}$$

③

In one minute  $\Rightarrow$  360 rev<sup>m</sup>.

$\Rightarrow$  60 sec.  $\longrightarrow$  360 rev<sup>m</sup>

1 sec.  $\longrightarrow$   $\left(\frac{360}{60}\right)$  rev<sup>m</sup>

1 sec.  $\longrightarrow$  6 rev<sup>m</sup>

$= (6 \times 2\pi)$  radian

$\rightarrow = \underline{(12\pi)^c}$

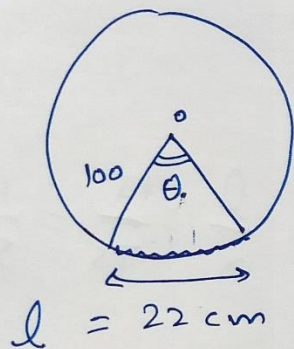


1 rev<sup>m</sup>  
 $= (2\pi)^c$

# Toppers Village

## Exercise 3.1 (class 11 Maths)

④



$$r = 100 \text{ cm}$$

radian

$$\theta = \frac{l}{r}$$

$$\theta = \left(\frac{22}{100}\right)^c$$

$$\text{Degree} = \frac{180}{\pi} \times (\text{Radian})$$

$$= \frac{180}{\left(\frac{22}{7}\right)} \times \frac{22}{100}$$

$$= \left(\frac{126}{10}\right)^{\circ}$$

$$= \left(\frac{126}{10}\right)^{\circ}$$

$$= (12.6)^{\circ}$$

$$= 12^{\circ} + \left(\frac{6}{10}\right)^{\circ}$$

$$= 12^{\circ} + \left(\frac{6}{10} \times 60\right)' = 12^{\circ} + (36)'$$

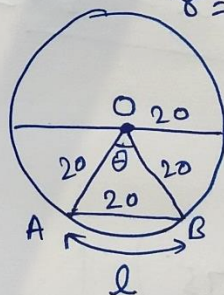
$$= 12^{\circ} 36'$$

⑤

$$d = 40 \text{ cm}$$

$$r = 20 \text{ cm}$$

$$\widehat{AB} = ?$$



$\triangle OAB \rightarrow$  Equilateral  $\triangle$

$$\text{minor arc} = l = r\theta$$

$$\Rightarrow l = 20 \times \frac{\pi}{3} = \frac{20\pi}{3}$$

$$l = r\theta$$

$$20 = 20 \times ?$$

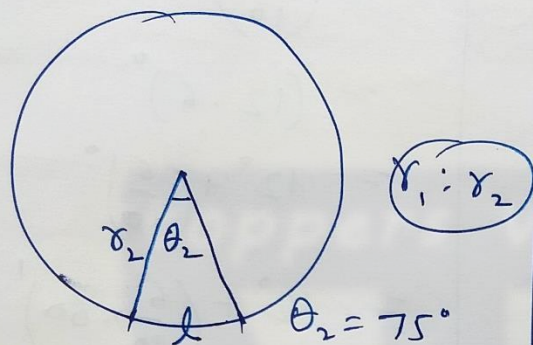
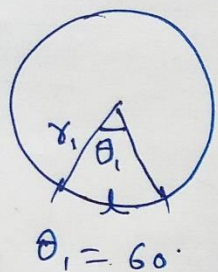
$$\theta = 60^{\circ}$$

~~$$\theta = 180$$~~

$$\theta = \frac{\pi}{180} \times 60$$

$$= \frac{\pi}{3}$$

6



length of arc = same = l

$l = r\theta$

$l = r_1 \theta_1$

$l = r_2 \theta_2$

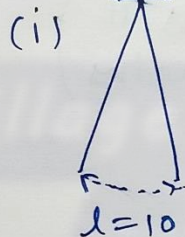
$r_1 \theta_1 = r_2 \theta_2$

$\Rightarrow r_1 (60) = r_2 (75)$

$\Rightarrow \frac{r_1}{r_2} = \frac{75}{60} = \frac{5}{4}$

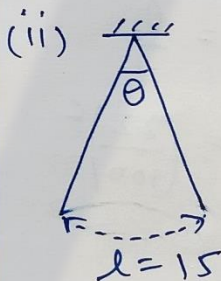
$r_1 : r_2 = 5 : 4$

7

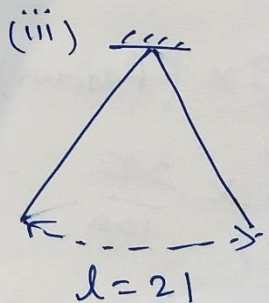


$r = 75 \text{ cm}$  radian  
 $\theta = ?$

radian  
 $\theta = \frac{l}{r} = \frac{10}{75} = \frac{2}{15}$



$\theta = \frac{l}{r} = \frac{15}{75} = \frac{1}{5}$



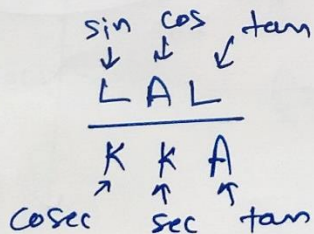
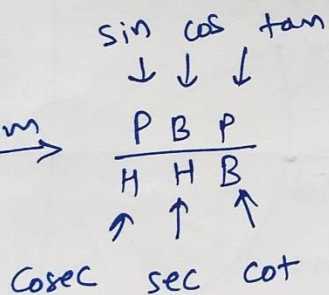
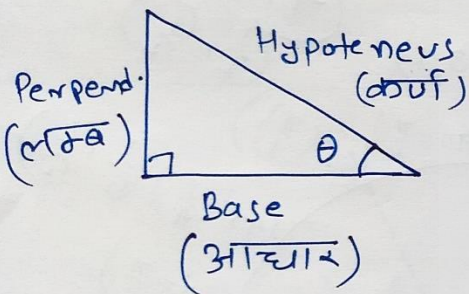
$\theta = \frac{l}{r} = \frac{21}{75} = \frac{7}{25}$

# Toppers Village

## Theory Before Exercise 3.2

### Trigonometric Functions

[Circular Functions]



### Identities

$$P^2 + B^2 = H^2$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x \rightarrow \sec x + \tan x = \frac{1}{\sec x - \tan x}$$

$$\operatorname{cosec}^2 x = 1 + \cot^2 x \rightarrow$$

### Trigonometric Table

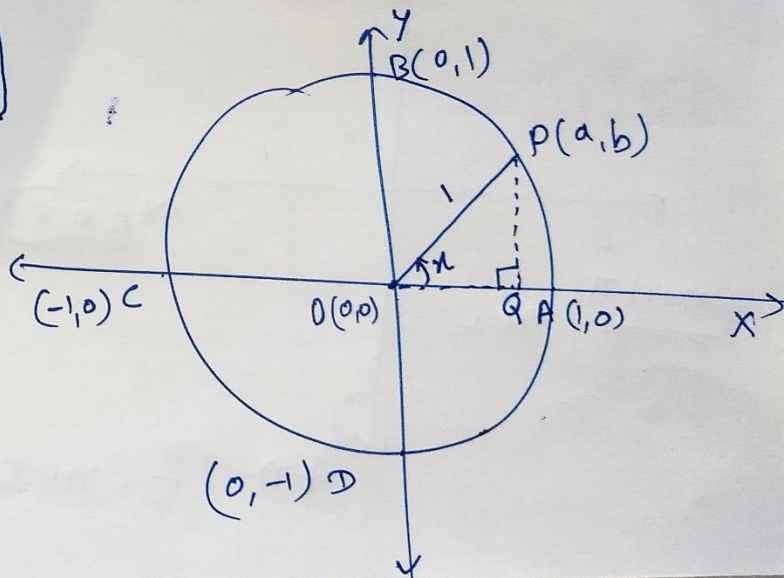
0°, 30°, 45°, 60°, 90°

↑ ↑ till 10<sup>th</sup> class ↑ ↑

### Now class -11, Trigonometric Fn<sup>n</sup>.

Unit Circle  
r = 1  
centre (0,0)

OP = 1  
PQ = b  
OQ = a



In  $\Delta OPQ$

$$\sin x = \frac{PQ}{OP} = \frac{b}{1} = b$$

$$\cos x = \frac{OQ}{OP} = \frac{a}{1} = a$$

$$P(a, b) \equiv P(\cos x, \sin x) \quad \star$$

Note  
 (1)  $OP^2 = OQ^2 + PQ^2$

$$\Rightarrow 1 = a^2 + b^2$$

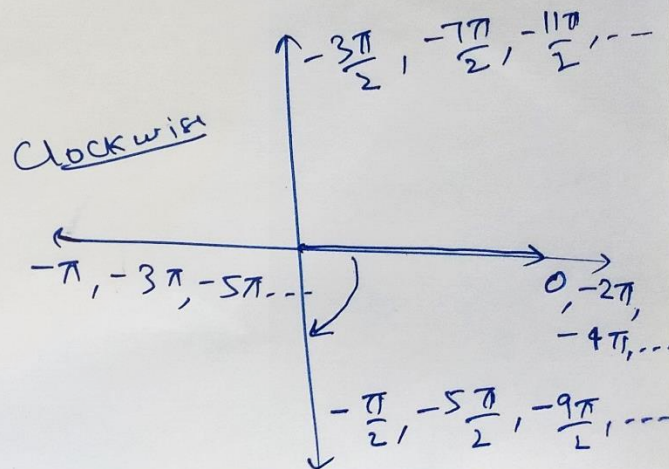
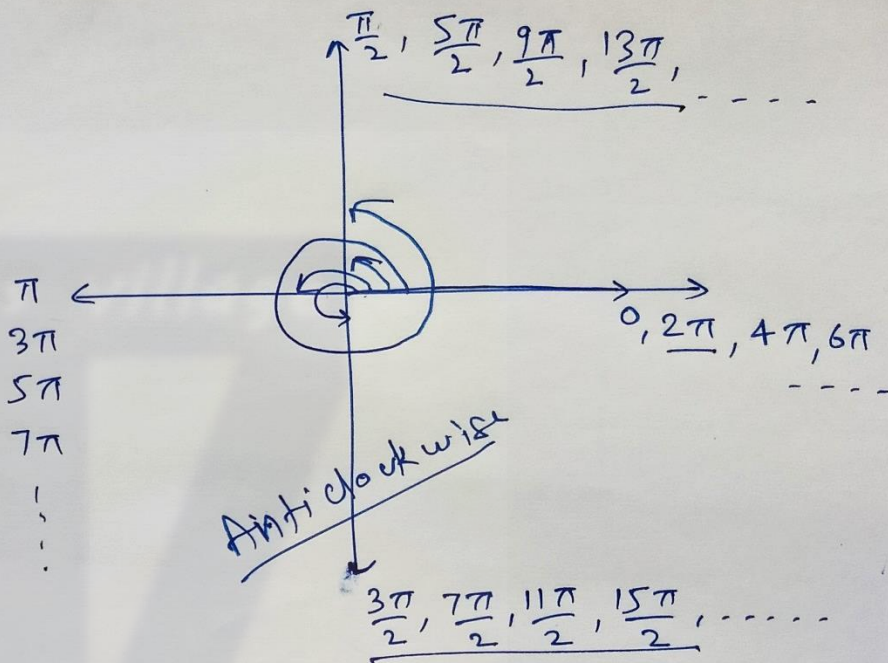
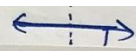
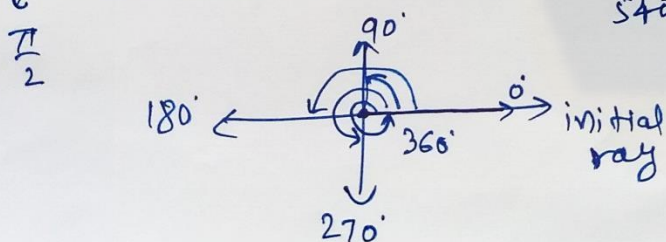
$$\Rightarrow \boxed{1 = \cos^2 x + \sin^2 x}$$

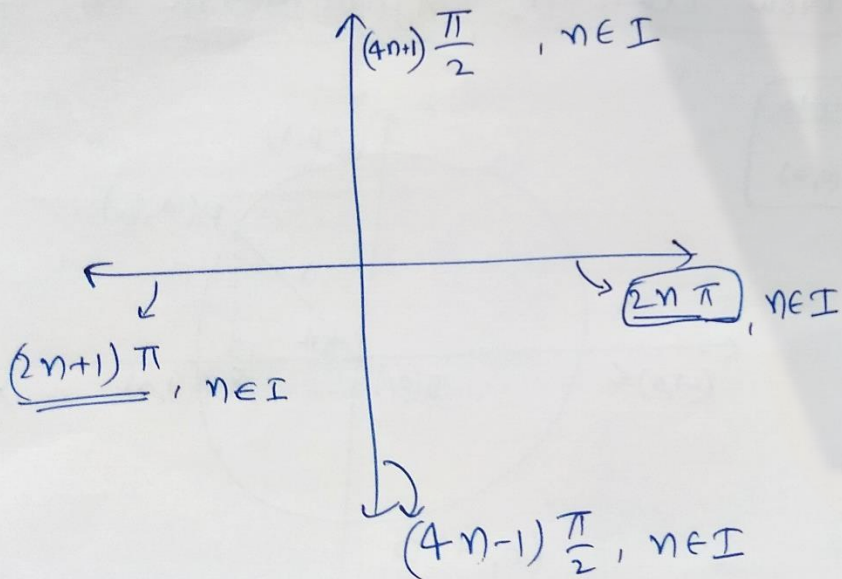
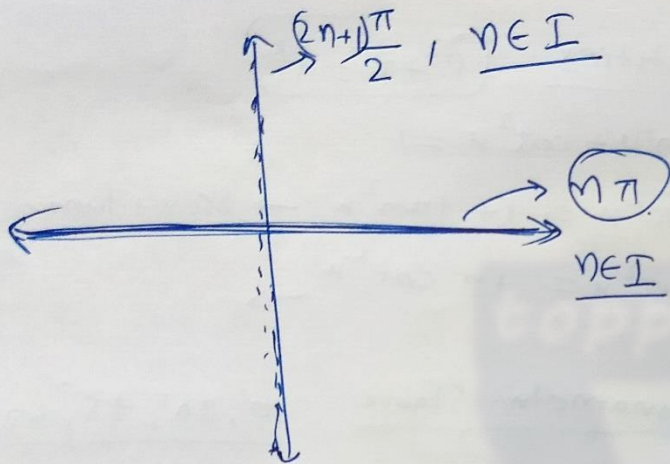
Note

Quadrantal Angles

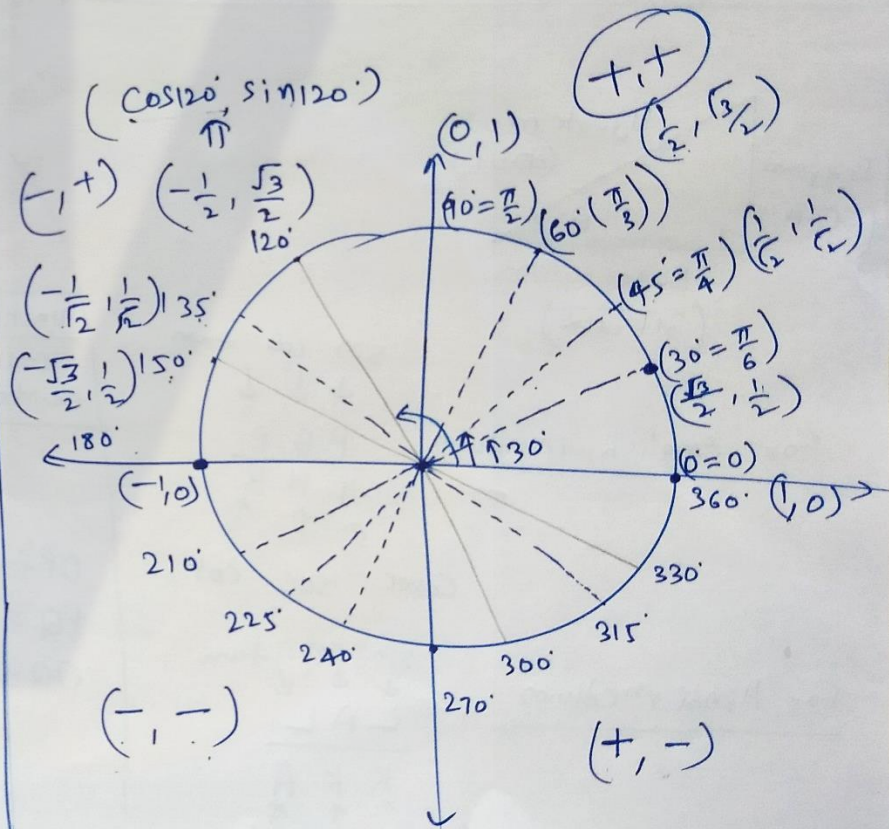
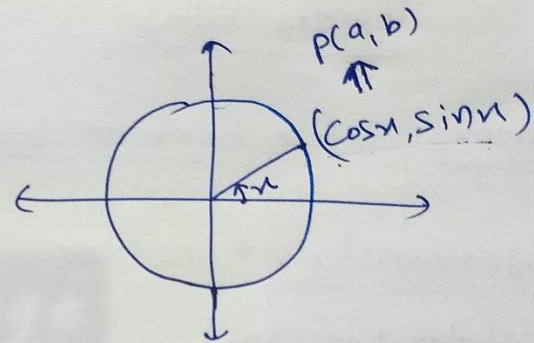
(2)

$90^\circ$  multiple =  $0, 90, \dots, 360, 450, 540, \dots$



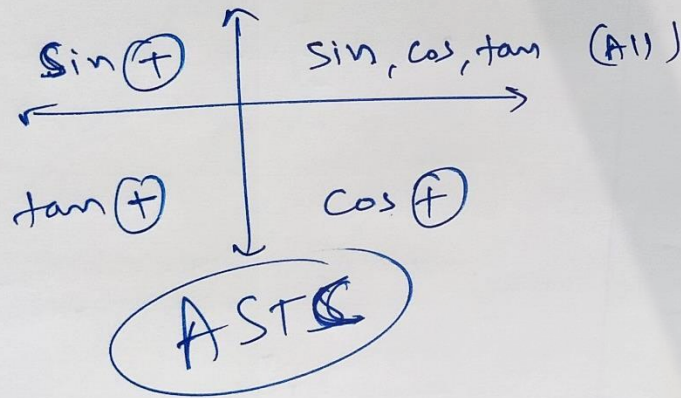
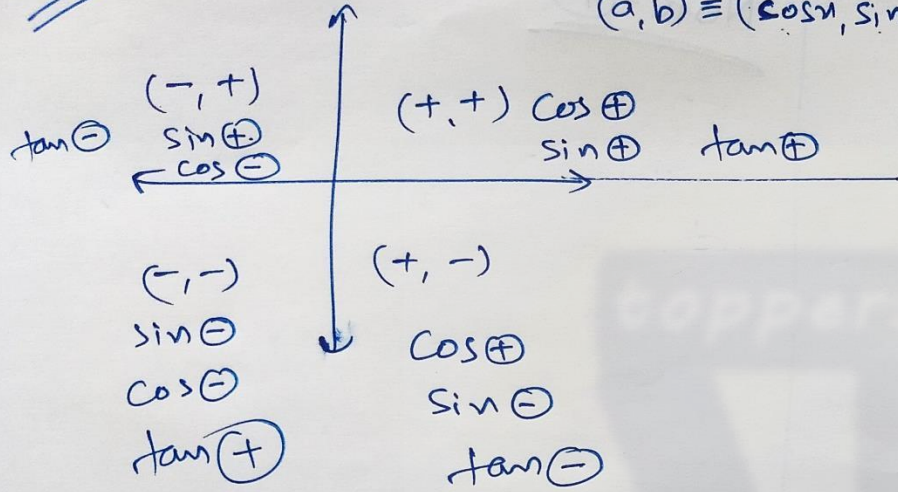


Note  
③

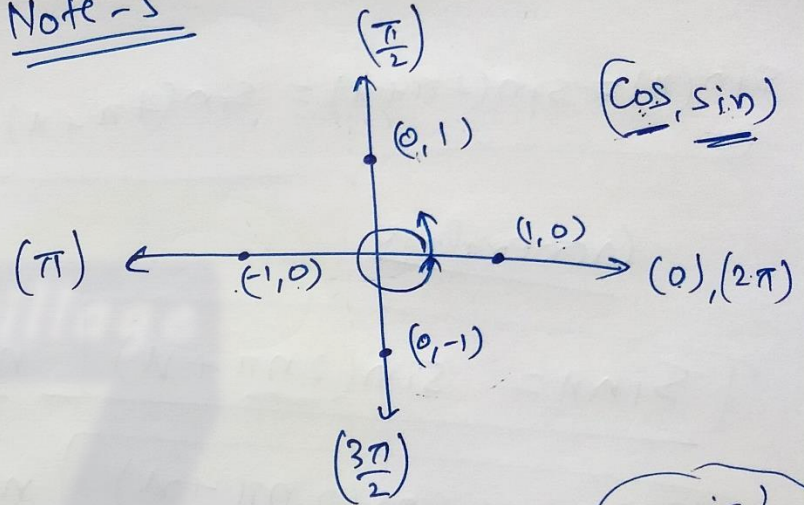


### Note - 4

Point  
 $(a, b) \equiv (\cos x, \sin x)$



### Note - 5



$$\begin{array}{l} \cos 0 = 1 \leftarrow \sin 0 = 0 \checkmark \\ \cos \frac{\pi}{2} = 0 \rightarrow \sin \frac{\pi}{2} = 1 \\ \cos \pi = -1 \leftarrow \sin \pi = 0 \checkmark \\ \cos \frac{3\pi}{2} = 0 \rightarrow \sin \frac{3\pi}{2} = -1 \\ \cos 2\pi = 1 \leftarrow \sin 2\pi = 0 \checkmark \end{array}$$

$$\begin{array}{l} \sin 30^\circ = \frac{1}{2} \\ \sin 390^\circ = \frac{1}{2} \end{array}$$

$$\sin(x) = \sin\left(\frac{2\pi}{\uparrow} + x\right)$$

One complete  
revolution

$$\sin(x) = \sin(2\pi + x) = \sin(4\pi + x) \dots$$

Generalize

$$\boxed{\sin x = \sin(2n\pi + x)} \quad n \in \mathbb{I}$$

$$\boxed{\cos x = \cos(2n\pi + x)} \quad n \in \mathbb{I}$$

Note: 6

$$\frac{\sin x = 0}{\sin n\pi = 0}, \quad \boxed{x = n\pi}, \quad n \in \mathbb{I}$$

$$\cos x = 0 \longrightarrow x = \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{I}$$

$$\longleftarrow \boxed{\cos\left(\frac{(2n+1)\pi}{2}\right) = 0}$$

# Toppers Village

Class - II - Maths

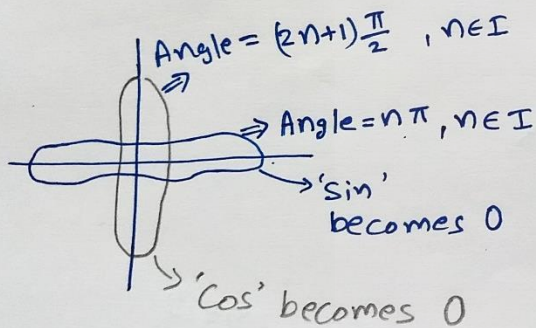
## Trigonometric Functions { Before Exercise 3.2 }

From Last Lecture

$\sin \oplus$	All $\oplus$
$\tan \oplus$	$\cos \oplus, \sec \oplus$

### Complementary

$\sin \rightarrow \cos$   
 $\cos \rightarrow \sin$   
 $\tan \rightarrow \cot$   
 $\cot \rightarrow \tan$   
 $\sec \rightarrow \operatorname{cosec}$   
 $\operatorname{cosec} \rightarrow \sec$



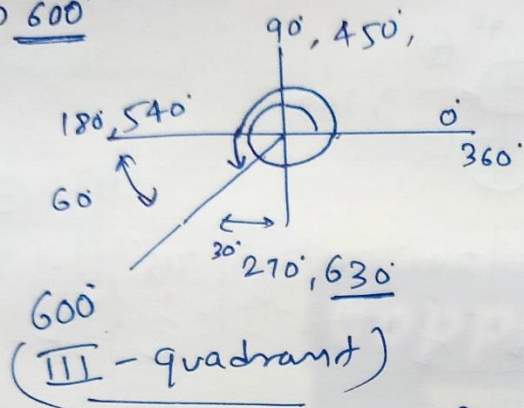
How to find values of trigonometric ratios values for Big Angles??

0, 30, 45, 60, 90

(i) Quadrant  $\rightarrow$  Final Answer  
 $\downarrow$   
Sign  $\rightarrow \oplus$   
 $\rightarrow \ominus$

(ii) Angle =  $90 \times n \pm \theta \approx \theta$   
90) Ang.  
Division Algorithm  
 $n$   $\rightarrow$  Even  $\rightarrow$  No complementary  
 $\rightarrow$  odd  $\rightarrow$  Complementary.

e.g. (i)  $\sin 600^\circ$



$$90 \overline{) 600} \begin{array}{r} 6 \\ - 540 \\ \hline 60 \end{array}$$

~~600~~  
~~90~~

$$600^\circ = 90 \times 6 + 60^\circ$$

$$\sin(\underbrace{600^\circ}_{\text{III}}) = \sin(\overbrace{90 \times 6 + 60^\circ}^{\text{III}})$$

↑  
Even

$$= -\sin 60^\circ$$

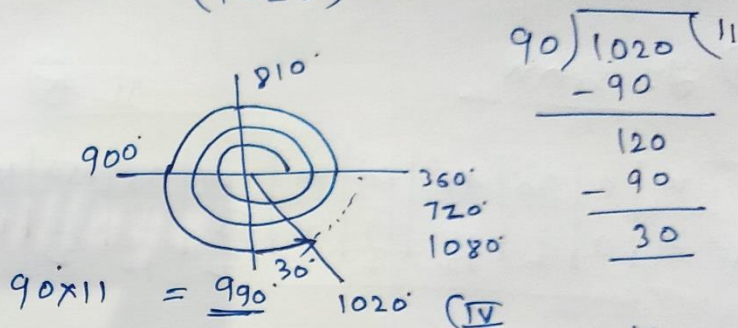
$$= -\frac{\sqrt{3}}{2}$$

$$\sin(600^\circ) = \sin(\overbrace{90 \times 7 - 30^\circ}^{\text{III}})$$

↑  
odd

$$= -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$$

(ii)  $\sec(1020^\circ)$



$$90 \overline{) 1020} \begin{array}{r} 11 \\ - 90 \\ \hline 120 \\ - 90 \\ \hline 30 \end{array}$$

$$90 \times 11 = 990$$

$$\sec(1020^\circ) = \sec(90 \times 11 + 30^\circ)$$

↑  
odd

$$= + \operatorname{cosec}(30^\circ)$$

$$\sec(1020^\circ) = +2$$

(iii)  $\tan(495^\circ)$

$$= \tan(90 \times 5 + 45^\circ)$$

↑  
odd

$$90 \overline{) 495} \begin{array}{r} 5 \\ - 450 \\ \hline 45 \end{array}$$

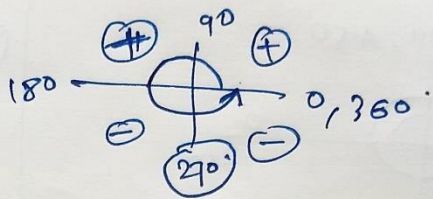
$$= -\cot(45^\circ)$$

$$= -(1)$$



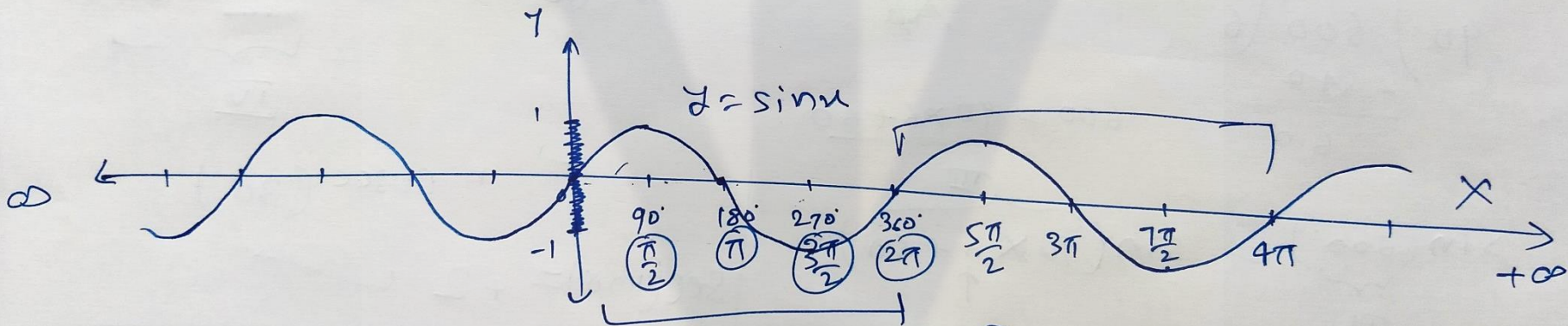
# Graph + Domain + Range

①  $y = \sin x$



$x$	0	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
$y$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0

I (+)      II (+)      III (-)      IV (-)

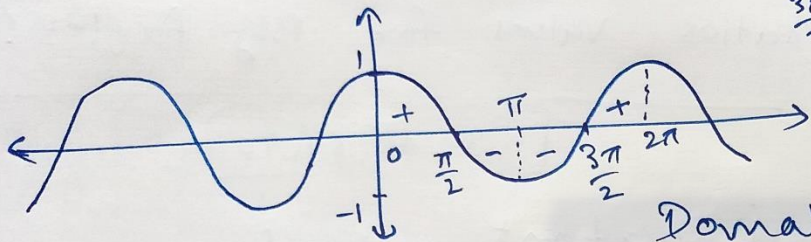
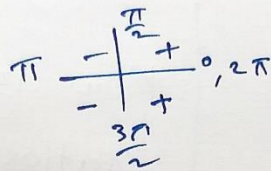


Repeat  
 $\leftarrow 2\pi \rightarrow$

Domain =  $(-\infty, \infty) = \mathbb{R}$   
 Range =  $[-1, 1]$  ✓

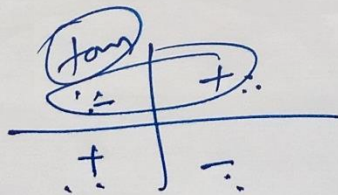
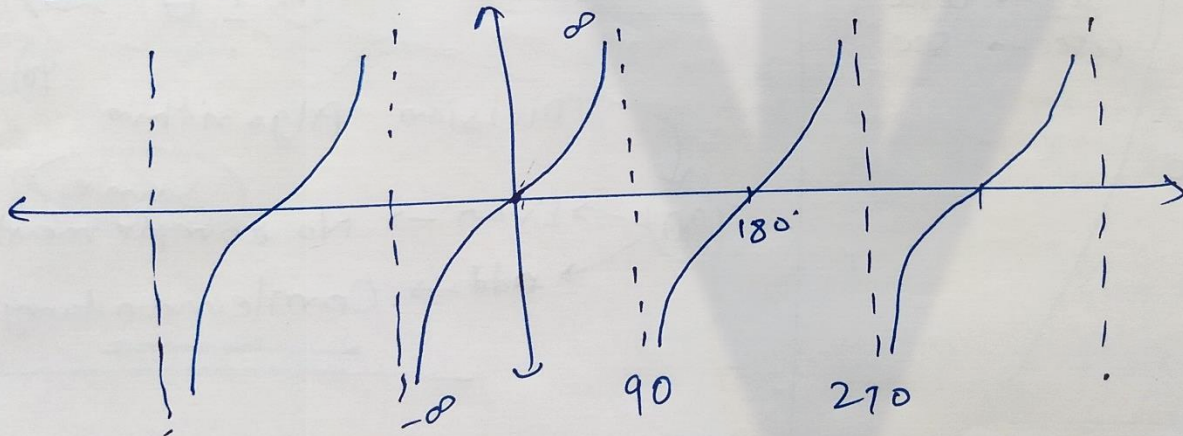
$-1 \leq \sin x \leq 1$  ★

②  $y = \cos x$



Domain =  $(-\infty, \infty) = \mathbb{R}$   
 Range =  $[-1, 1]$

③  $y = \tan x = \frac{\sin x}{\cos x} \neq 0$   $x \in \mathbb{R} - \left\{ a : a = (n+1)\frac{\pi}{2} \right\}$

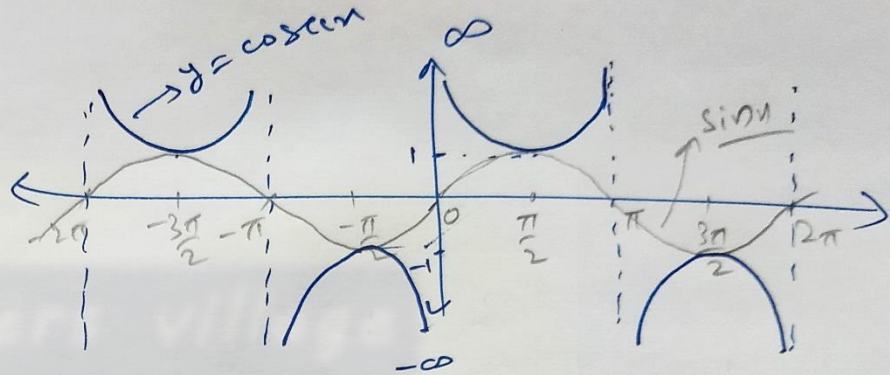


Domain =  $\mathbb{R} - \left\{ x : x = (2n+1)\frac{\pi}{2} \right\}$   $n \in \mathbb{I}$   
 Range =  $(-\infty, \infty) = \mathbb{R}$

④  $y = \operatorname{cosec} x = \frac{1}{\sin x} \neq 0$

Domain:  $\mathbb{R} - \{x : x = n\pi, n \in \mathbb{I}\}$

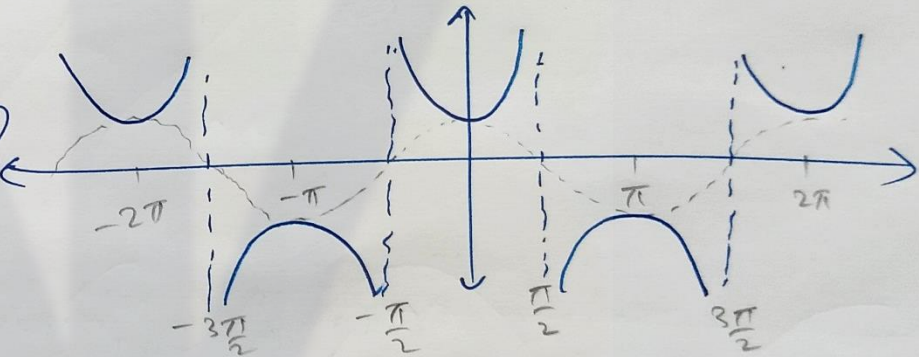
Range:  $(-\infty, -1] \cup [1, \infty)$



⑤  $y = \sec x = \frac{1}{\cos x} \neq 0$

Domain =  $\mathbb{R} - \left\{x : x = (2n+1)\frac{\pi}{2}\right\}$

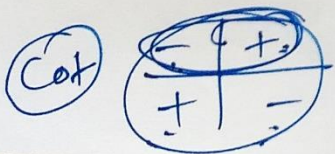
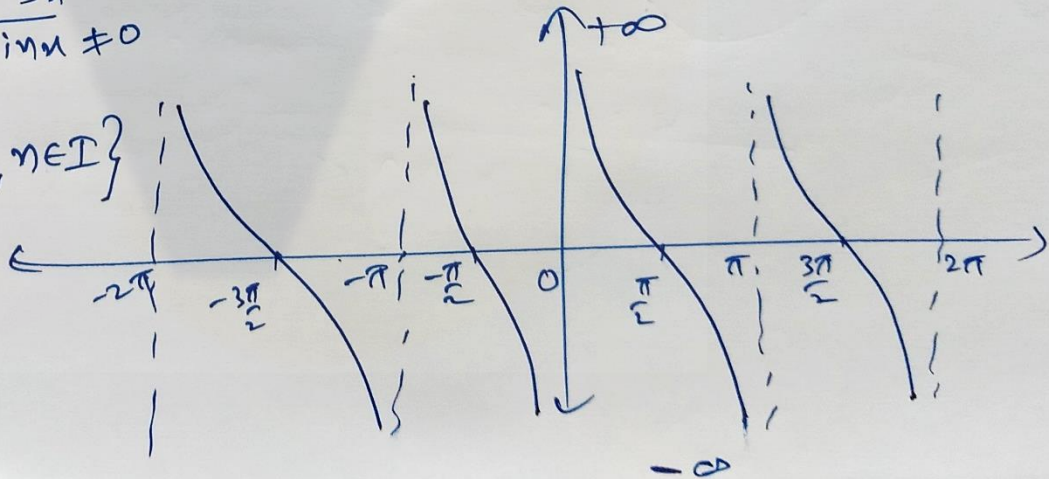
Range =  $(-\infty, -1] \cup [1, \infty)$



⑥  $y = \tan x = \frac{\sin x}{\cos x} \neq 0$

Domain =  $\mathbb{R} - \{x : x = n\pi, n \in \mathbb{I}\}$

Range =  $(-\infty, \infty) = \mathbb{R}$



# Toppers Village

Class - 11 - Maths

## Exercise 3.2

①  $\cos x = -\frac{1}{2}$ ,  $x \rightarrow$  II quadrant

$\sin x$	$\tan x$	$\cot x$	$\sec x$	$\operatorname{cosec} x$
↓	+	+	↓	↓
⊖	⊕	⊕	⊖	⊖

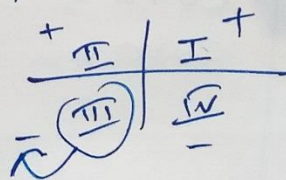
$$\underline{\sin^2 x} + \underline{\cos^2 x} = 1 \quad \checkmark$$

$$\Rightarrow \sin^2 x + \left(-\frac{1}{2}\right)^2 = 1 \quad \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\Rightarrow \sin^2 x = 1 - \frac{1}{4}$$

$$\Rightarrow \sin^2 x = \frac{4-1}{4} = \frac{3}{4}$$

$$\boxed{\sin x = \pm \frac{\sqrt{3}}{2}}$$



$$\sin x = -\frac{\sqrt{3}}{2}, \quad \cos x = -\frac{1}{2}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{+\frac{\sqrt{3}}{2}}{+\frac{1}{2}} = \sqrt{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\frac{1}{2}\right)} = -2$$

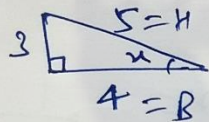
$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}$$

$$\textcircled{2} \quad \sin x = \frac{3}{5}, \quad x \rightarrow \text{II}$$

sin	+
tan	+
cos	-

$\cos x$ ,  $\tan x$ ,  $\cot x$ ,  $\operatorname{cosec} x$ ,  $\sec x$   
 - - -  $\oplus$  -

$$\sin x = \frac{3}{5} = \frac{P}{H}$$



$$\cos x = \frac{B}{H} = -\frac{4}{5}$$

$$\tan x = \frac{P}{B} = -\frac{3}{4}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{5}{3}$$

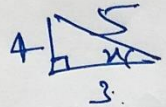
$$\sec x = \frac{1}{\cos x} = -\frac{5}{4}$$

~~$$\tan x =$$~~

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}$$

$$\textcircled{3} \quad \cot x = \frac{3}{4}, \quad x \rightarrow \text{III}$$

$$\cot x = \frac{3}{4} = \frac{B}{P}$$



$$\sin x = \frac{P}{H} = -\frac{4}{5}$$

$$\cos x = \frac{B}{H} = -\frac{3}{5}$$

$$\tan x = \frac{4}{3}$$

$$\sec x = -\frac{5}{3}$$

$$\operatorname{cosec} x = -\frac{5}{4}$$

$$\textcircled{4} \quad \sec x = \frac{13}{5} = \frac{H}{B}, \quad x \rightarrow \text{IV}$$

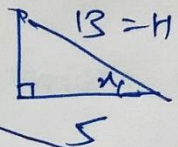
$$\cos x = \frac{5}{13}$$

$$\sin x = \frac{P}{H} = -\frac{12}{13} = -\frac{12}{13}, \quad P=12$$

$$\tan x = -\frac{12}{5}$$

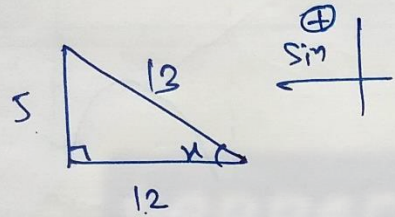
$$\cot x = -\frac{5}{12}$$

$$\operatorname{cosec} x = -\frac{13}{12}$$



⑤  $\tan u = -\frac{5}{12} = \frac{P}{B}$   $u \rightarrow \text{II}$

$\cot u = -\frac{12}{5}$   
 $\sin u = \frac{5}{13}$   
 $\cos u = -\frac{12}{13}$   
 $\sec u = -\frac{13}{12}$   
 $\operatorname{cosec} u = \frac{13}{5}$

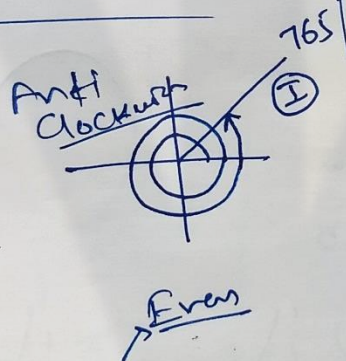


⑥  $\sin(765^\circ)$   
 $765 = 90 \times n + \theta$  quadrant

$$90 \overline{) 765} \begin{matrix} 8 \\ -720 \\ \hline 45 \end{matrix}$$

$765 = 90 \times 8 + 45$

$\sin(765^\circ) = \sin(90 \times 8 + 45^\circ)$   
 $= +\sin(45^\circ)$   
 $= \frac{1}{\sqrt{2}}$



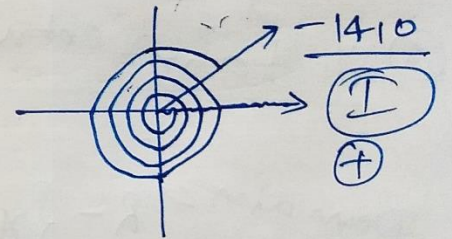
⑦  $\operatorname{cosec}(-1410^\circ)$   
 $90 \times n + \theta$  quadrant

$$90 \overline{) 1410} \begin{matrix} 15 \\ -90 \\ \hline 510 \\ -450 \\ \hline 60 \end{matrix}$$

$1410 = 90 \times 15 + 60$

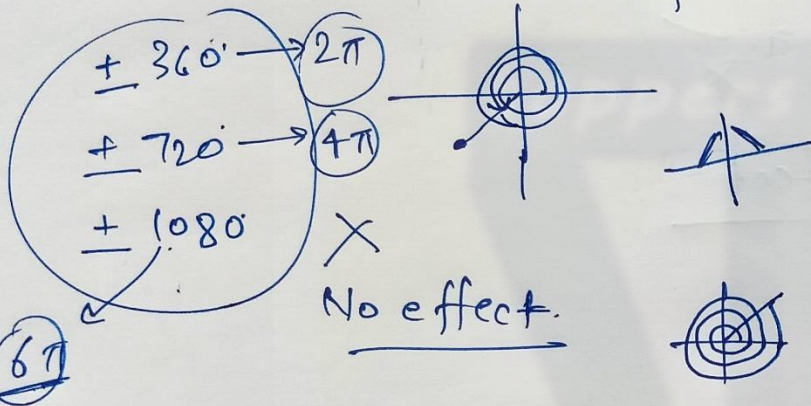
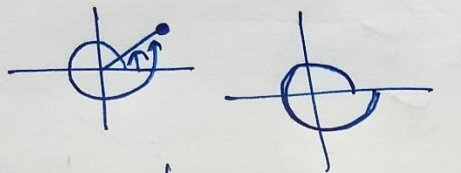
Angle =  $-1410 = -90 \times 15 - 60$

Clock



$\operatorname{cosec}(-1410^\circ)$   
 $= \operatorname{cosec}(-90 \times 15 - 60^\circ)$   
 $= +\sec(60^\circ)$   
 $= 2$

$$\textcircled{8} \tan\left(\frac{19\pi}{3}\right)$$



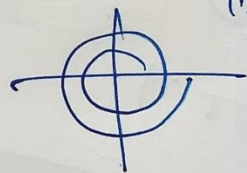
$$\begin{aligned} \tan\left(\frac{19\pi}{3}\right) &= \tan\left(\textcircled{6\pi} + \frac{\pi}{3}\right) \\ &= \tan\left(\frac{\pi}{3}\right) \\ &= \sqrt{3} \quad \checkmark \end{aligned}$$

$$\textcircled{9} \sin\left(-\frac{11\pi}{3}\right) \begin{array}{l} \leftarrow \text{Degrees} \\ \uparrow \text{Radian} \end{array} \quad \frac{\pi \rightarrow 180^\circ}{\text{---}}$$

$$= \sin\left(-\frac{11 \times 180^\circ}{3}\right)$$

$$= \sin(-660^\circ) \quad \text{---}$$

$$= \sin(+720^\circ - 660^\circ) \quad \begin{array}{l} 720^\circ = 2 \times 360^\circ \\ \text{---} \end{array}$$



$$\begin{aligned} &= \sin(60^\circ) \\ &= \frac{\sqrt{3}}{2} \quad \checkmark \end{aligned}$$

$$(10) \cot\left(\frac{-15\pi}{4}\right)$$

$$= \cot\left(\frac{-16\pi + \pi}{4}\right)$$

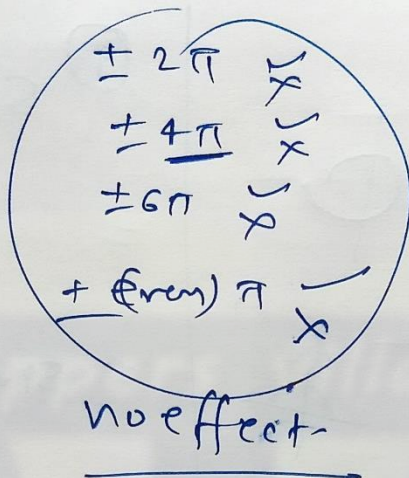
$$= \cot\left(-\frac{16\pi}{4} + \frac{\pi}{4}\right)$$

$$= \cot\left(\cancel{-4\pi} + \frac{\pi}{4}\right)$$

$$= \cot\left(\frac{\pi}{4}\right)$$

$$= \cot(45^\circ)$$

$$= 1$$



Remove

# Toppers Village

## Chapter: 3 Trigonometric Functions (Class 11)

All formulae (Before Exercise 3.3)

+

Examples.

### ① Effect of Negative Angle.

$$\sin(-x) = -\sin x$$

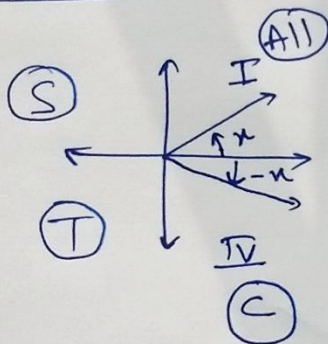
$$\boxed{\cos(-x) = \cos x}$$

$$\tan(-x) = -\tan x$$

$$\operatorname{cosec}(-x) = -\operatorname{cosec} x$$

$$\boxed{\sec(-x) = \sec x}$$

$$\cot(-x) = -\cot x$$



② ★  $\square\square \pm \square\square$

$$\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$$

$$\cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$$

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$$\sin(x-y) = \sin x \cdot \cos y - \cos x \cdot \sin y$$

③ Multiply to Add / subtract  
(x) (+, -)

$$2 \sin x \cdot \cos y = \sin(x+y) + \sin(x-y)$$

$$2 \cos x \cdot \sin y = \sin(x+y) - \sin(x-y)$$

$$2 \cos x \cdot \cos y = \cos(x+y) + \cos(x-y)$$

$$2 \sin x \cdot \sin y = \cos(x-y) - \cos(x+y)$$

③  $\cot \frac{1}{2}$

④  $\oplus$  or  $\ominus \longrightarrow \otimes$

Add/subtract to multiply.

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \sin\left(\frac{x-y}{2}\right) \cdot \cos\left(\frac{x+y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)$$

$$\star \cos x - \cos y = 2 \sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{y-x}{2}\right)$$

$$= \underline{\underline{-2 \sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)}}$$

$$\textcircled{5} \quad \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \checkmark$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \checkmark$$

$$\star \cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

$$\star \cot(x-y) = \frac{\cot x \cot y + 1}{-\cot x + \cot y}$$

## ⑥ Multiple Angles.

$$\begin{cases} \sin 2x = 2 \sin x \cdot \cos x = \frac{2 \sin x \cdot \cos x}{1} = \frac{(2 \sin x \cos x)}{(\sin^2 x + \cos^2 x)} / \cos^2 x = \frac{2 \tan x}{1 + \tan^2 x} \\ \sin 2x = \frac{2 \tan x}{1 + \tan^2 x} \end{cases}$$

$$\begin{cases} \cos 2x = \cos^2 x - \sin^2 x \\ \cos 2x = 2 \cos^2 x - 1 \\ \cos 2x = 1 - 2 \sin^2 x \\ \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x} \end{cases}$$

$$\begin{cases} \tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \end{cases}$$

$$\begin{cases} \cos^2 x = 1 - \sin^2 x \\ \sin^2 x = 1 - \cos^2 x \end{cases}$$

$$\sin 3x = \sin(x+2x)$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$

Trick

$$31 - 43$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

$$43 - 31$$

$$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

Application.

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\sin\left(\frac{\pi}{2} + u\right) = \cos u$$

$$\cos\left(\frac{\pi}{2} + u\right) = -\sin u$$

$$\sin(\pi - u) = \sin u$$

$$\cos(\pi - u) = -\cos u$$

$$\sin(\pi + u) = -\sin u$$

$$\cos(\pi + u) = -\cos u$$

$$\sin\left(\frac{3\pi}{2} - u\right) = -\cos u$$

$$\cos\left(\frac{3\pi}{2} - u\right) = -\sin u$$

$$\sin(2\pi - u) = -\sin u$$

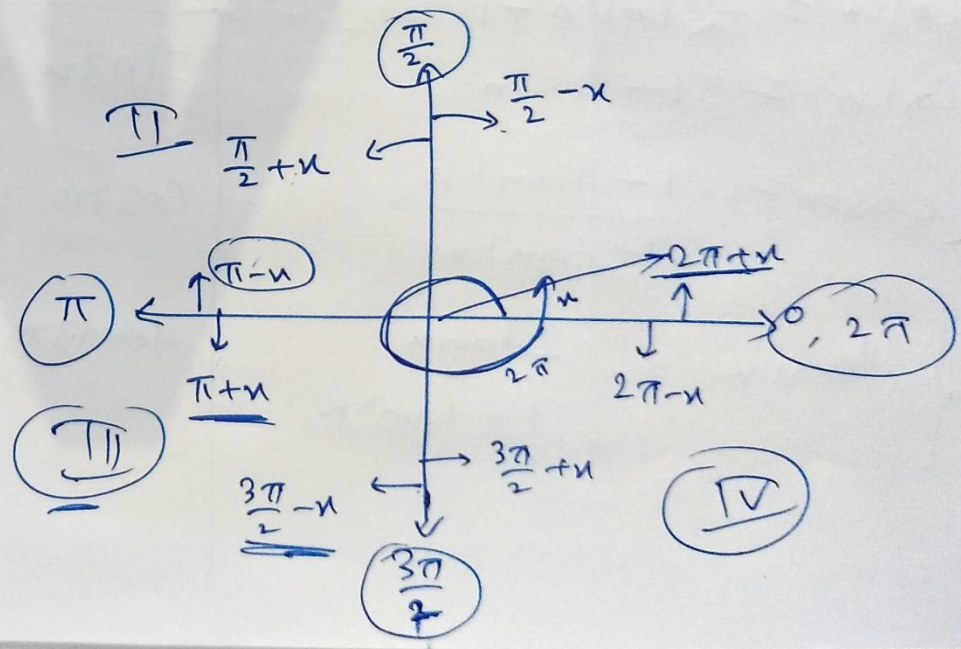
$$\cos(2\pi - u) = +\cos u$$

$$\sin(2\pi + u) = \sin u$$

$$\cos(2\pi + u) = \cos u$$

$$\sin\left(\frac{3\pi}{2} + u\right) = ?$$

$$\cos\left(\frac{3\pi}{2} + u\right) = ?$$



5 (i)  $\sin 75^\circ$

Examples

(11)  $\sin 15^\circ$   
 $= \sin (45^\circ - 30^\circ)$   
 $= \sin (45^\circ) \cos (30^\circ) - \cos (45^\circ) \sin (30^\circ)$   
 $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$   
 $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x}$$

$$\Rightarrow \tan 3x = \tan 2x \cdot \tan 2x \cdot \tan x$$

$$= \tan 2x + \tan x$$

$$\Rightarrow \tan 3x - \tan 2x - \tan x$$

$$= \tan 3x \cdot \tan 2x \cdot \tan x$$

(14) Prove

$$\tan 3x \cdot \tan 2x \cdot \tan x$$

$$= \tan 3x - \tan 2x - \tan x$$

$3x = (2x + x)$

$\tan \rightarrow$

$$\tan 3x = \tan(2x + x)$$

$\sec^2 \pi = 2(\sqrt{2})$

$\dots = 10$

Example, (17)

Prove that

$$\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$$

$$\text{LHS} = \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x}$$

$$= \frac{2 \sin \left(\frac{6x}{2}\right) \cdot \cos \left(\frac{4x}{2}\right) - 2\sin 3x}{-2 \sin \left(\frac{5x+x}{2}\right) \cdot \sin \left(\frac{5x-x}{2}\right)}$$

$$= \frac{\cancel{2} \sin 3x \cdot \cos 2x - \cancel{2} \sin 3x}{-\cancel{2} \sin 3x \cdot \sin 2x}$$

$$= \frac{\cos 2x - 1}{-\sin 2x} = \frac{1 - \cos 2x}{\sin 2x}$$

$$= \frac{1 - \cos 2x}{\sin 2x}$$

Annotations for the above expression:

- $\cos^2 x - \sin^2 x$
- $2\cos^2 x - 1$
- $1 - 2\sin^2 x$
- $\frac{1 - \tan^2 x}{1 + \tan^2 x}$
- $\frac{2 \sin x \cos x}{2 \tan x}$
- $\frac{2 \tan x}{1 + \tan^2 x}$

$$= \frac{1 - (1 - 2\sin^2 x)}{2 \sin x \cos x}$$

$$= \frac{x - x + 2\sin^2 x}{2 \sin x \cdot \cos x}$$

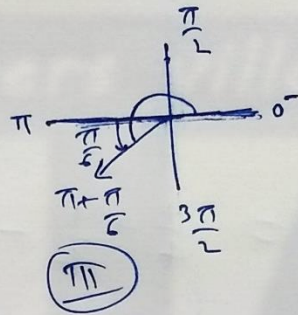
$$= \frac{\sin x}{\cos x}$$

$$= \tan x = \text{RHS.}$$

# Toppers Village

## Class-11. Maths. Exercise 3.3

$$\operatorname{cosec} \frac{7\pi}{6} = \operatorname{cosec} \left( \overbrace{\pi + \frac{\pi}{6}}^{\text{III}} \right)$$



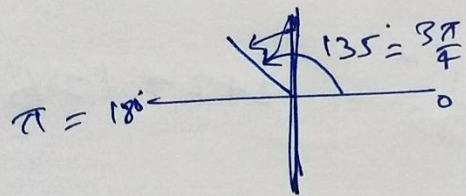
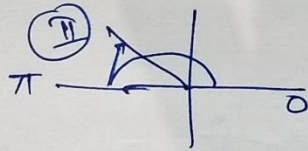
$$= -\operatorname{cosec} \left( \frac{\pi}{6} \right) = -2$$

$$\operatorname{cosec} \frac{5\pi}{6}$$

$$\sin \frac{\pi}{6} = \sin 30^\circ = \frac{1}{2}$$

$$\sin \frac{3\pi}{4}$$

$$\operatorname{cosec} \left( \pi - \frac{\pi}{6} \right) = +\operatorname{cosec} \left( \frac{\pi}{6} \right) = 2$$



$$\sin \left( \frac{3\pi}{4} \right) = \sin \left( \overbrace{\frac{\pi}{2} + \frac{\pi}{4}}^{\text{II}} \right) = +\cos \left( \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}}$$

$$\sin \left( \frac{\pi}{2} + u \right)$$

$$\operatorname{cosec} \frac{7\pi}{6} = -2$$

$$\operatorname{cosec} \frac{5\pi}{6} = +2$$

$$\sin \frac{3\pi}{4} = +\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{6} = 30^\circ$$

$$\frac{\pi}{4} = 45^\circ$$

$$\frac{\pi}{3} = 60^\circ$$

$$\textcircled{1} \quad \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = \frac{1}{4} + \frac{1}{4} - 1 = \frac{1}{2} - 1 = -\frac{1}{2} \quad \checkmark$$

$$\textcircled{2} \quad 2 \sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cdot \cos^2 \frac{\pi}{3} = 2 \left(\frac{1}{4}\right) + (-2)^2 \cdot \left(\frac{1}{4}\right) = \frac{1}{2} + 4 \times \frac{1}{4} = \frac{3}{2} \quad \checkmark$$

$$\textcircled{3} \quad \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} = (\sqrt{3})^2 + 2 + 3 \left(\frac{1}{3}\right) = 3 + 2 + 1 = 6 \quad \checkmark$$

$$\begin{aligned} \textcircled{4} \quad 2 \sin^2 \left(\frac{3\pi}{4}\right) + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} &= 2 \left(\frac{1}{2}\right) + 2 \left(\frac{1}{2}\right) + 2(4) = \\ &= 1 + 1 + 8 = 10 \quad \checkmark \end{aligned}$$

$$\textcircled{5} \quad (i) \quad \sin 75^\circ$$

$$= \sin(45^\circ + 30^\circ)$$

$$\sin(x+y) = \sin x \cdot \cos y + \sin y \cdot \cos x$$

$\begin{matrix} \nearrow & \nearrow \\ \uparrow & \uparrow \\ 45^\circ & 30^\circ \end{matrix} \quad \begin{matrix} \nearrow & \nearrow \\ \uparrow & \uparrow \\ 30^\circ & 45^\circ \end{matrix}$

$$\sin(75^\circ) = \sin 45^\circ \cdot \cos 30^\circ + \sin 30^\circ \cdot \cos 45^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \quad \checkmark$$

$$(ii) \quad \tan 15^\circ$$

$$15 = 45 - 30 \quad \checkmark$$

$$15 = 60 - 45 \quad \checkmark$$

$$\tan(15^\circ) = \tan(45^\circ - 30^\circ)$$

$\textcircled{x} \quad \textcircled{y}$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$$

$$x = 45$$

$$y = 30$$

$$\tan 15^\circ = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Exercise 3.3 Q6 - Q11 (~~use~~ Formulae)

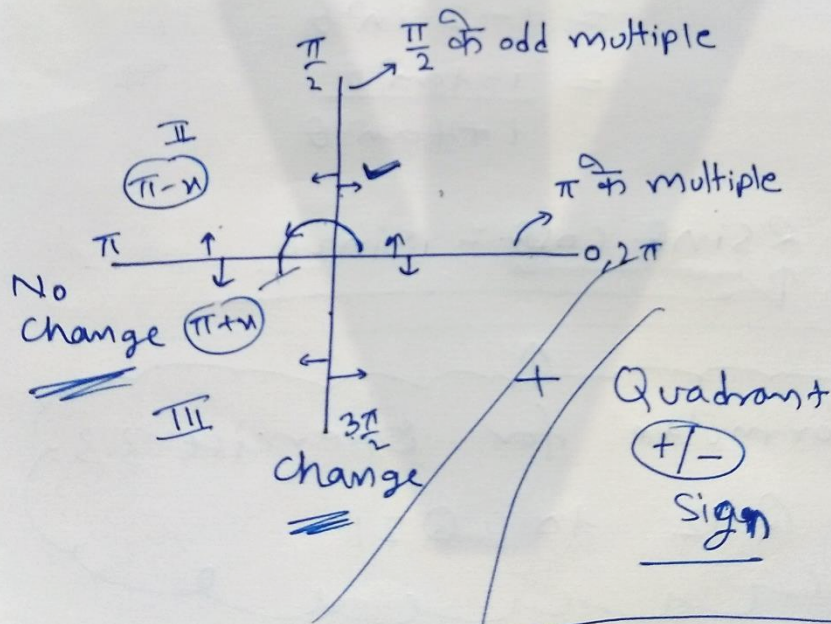
$$\cos A \cos B - \sin A \sin B = \cos(A+B)$$

$$\cos A \cos B + \sin A \sin B = \cos(A-B)$$

$$\sin A \cos B + \cos A \sin B = \sin(A+B)$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$



$\sin(-x) = -\sin x$   
 $\cos(-x) = \cos x$

# Toppers Village

## Class 11 Maths × Exercise 3.3

$$\textcircled{6} \quad \cos\left(\frac{\pi}{4} - x\right) \cdot \cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right) \cdot \sin\left(\frac{\pi}{4} - y\right) \\ = \sin(x+y)$$

$$\text{L.H.S.} = \underbrace{\cos\left(\frac{\pi}{4} - x\right)}_A \cdot \underbrace{\cos\left(\frac{\pi}{4} - y\right)}_B - \underbrace{\sin\left(\frac{\pi}{4} - x\right)}_A \cdot \underbrace{\sin\left(\frac{\pi}{4} - y\right)}_B$$

$$\cos A \cos B - \sin A \sin B = \cos(A+B)$$

$$= \cos\left(\frac{\pi}{4} - x + \frac{\pi}{4} - y\right)$$

$$= \cos\left(\frac{\pi}{2} - \underbrace{(x+y)}_{\theta}\right)$$

$$= \sin(x+y)$$

$$= \text{RHS.}$$

$$\left(\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right)$$

$$\textcircled{7} \quad \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

$$\text{L.H.S.} = \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)}$$

$$= \frac{\left(\tan\frac{\pi}{4} + \tan x\right)}{1 - \tan\frac{\pi}{4} \cdot \tan x}$$

$$\frac{\left(\tan\frac{\pi}{4} - \tan x\right)}{1 + \tan\frac{\pi}{4} \cdot \tan x}$$

$$\boxed{\tan\frac{\pi}{4} = \tan 45^\circ = 1}$$

$$= \frac{\left(\frac{1 + \tan x}{1 - \tan x}\right)}{\left(\frac{1 - \tan x}{1 + \tan x}\right)} = \frac{(1 + \tan x)^2}{(1 - \tan x)^2} = \text{RHS.}$$

$$\textcircled{8} \quad \frac{\cos(\pi+x) \cdot \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$$

$$\text{LHS} = \frac{\cos(\pi+x) \cdot \cos(-x)}{\sin(\pi-x) \cdot \cos\left(\frac{\pi}{2}+x\right)}$$

10<sup>th</sup> class  $\cos(90-\theta) = \sin\theta$

$\hookrightarrow \cos\left(\frac{\pi}{2}-x\right) = \sin x$

$\cos(\pi+x) = -\cos x$  (III)

$\sin(\pi-x) = +\sin x$  (II)

$\cos\left(\frac{\pi}{2}+x\right) = -\sin x$

$$\text{LHS} = \frac{(-\cos x) \cdot (\cos x)}{(\sin x) \cdot (-\sin x)}$$

$$= \frac{\cos^2 x}{\sin^2 x} = \cot^2 x = \text{RHS}$$

$$\textcircled{9} \quad \cos\left(\frac{3\pi}{2}+x\right) \cdot \cos(2\pi+x) \cdot \left[\cot\left(\frac{3\pi}{2}-x\right) + \cot(2\pi+x)\right]$$

$$\cos\left(\frac{3\pi}{2}+x\right) = +\sin x \quad \text{(IV)}$$

$$\cos(2\pi+x) = +\cos x$$

$$\cot\left(\frac{3\pi}{2}-x\right) = +\tan x \quad \text{(III)}$$

$$\cot(2\pi+x) = +\cot x$$

$$\text{LHS} = \sin x \cdot \cos x \cdot [\tan x + \cot x]$$

$$= \sin x \cdot \cos x \left[ \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right]$$

$$= \cancel{\sin x} \cdot \cancel{\cos x} \left[ \frac{\sin^2 x + \cos^2 x}{\cancel{\cos x} \cdot \cancel{\sin x}} \right]$$

$$= 1 = \text{RHS} \quad \checkmark$$

$$\textcircled{10} \sin(n+1)x \cdot \sin(n+2)x + \cos(n+1)x \cdot \cos(n+2)x = \cos x$$

$$\text{LHS.} = \frac{\sin(n+1)x}{A} \cdot \frac{\sin(n+2)x}{B} + \frac{\cos(n+1)x}{A} \cdot \frac{\cos(n+2)x}{B}$$

$$\boxed{\sin A \cdot \sin B + \cos A \cdot \cos B = \cos(A-B)}$$

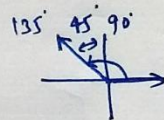
$$= \cos \left( \overset{(n+1)x}{\uparrow} A - \overset{(n+2)x}{\uparrow} B \right)$$

$$= \cos((n+1)x - (n+2)x) = \cos(\cancel{nx} + x - \cancel{nx} - 2x) = \cos(-x) = \cos x = \underline{\underline{\text{RHS}}}$$

$$\textcircled{11} \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$$

$$\text{LHS.} = \underbrace{\cos\left(\frac{3\pi}{4} + x\right)}_{\cos(A+B)} - \underbrace{\cos\left(\frac{3\pi}{4} - x\right)}_{\cos(A-B)}$$

$$\begin{aligned} \cos \frac{3\pi}{4} &= -\frac{1}{\sqrt{2}} \\ \sin \frac{3\pi}{4} &= \frac{1}{\sqrt{2}} \end{aligned}$$



$$2 = \sqrt{2} \cdot \sqrt{2}$$

$$= \left[ \cancel{\cos \frac{3\pi}{4}} \cdot \cos x - \underbrace{\sin \frac{3\pi}{4}}_{\frac{1}{\sqrt{2}}} \cdot \sin x \right] - \left[ \cancel{\cos \frac{3\pi}{4}} \cdot \cos x + \sin \frac{3\pi}{4} \cdot \sin x \right]$$

$$= -\frac{1}{\sqrt{2}} \cdot \sin x - \frac{1}{\sqrt{2}} \cdot \sin x = -2\left(\frac{1}{\sqrt{2}} \cdot \sin x\right) = -\sqrt{2} \sin x = \text{RHS.}$$

# Toppers Village

## Class - 11 - Maths \* Exercise 3.3

(12)  $\sin^2 6x - \sin^2 4x = \sin 2x \cdot \sin 10x$

LHS =  $\sin^2 6x - \sin^2 4x$   $a^2 - b^2$

=  $(\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

=  $\left[ 2 \sin\left(\frac{6x+4x}{2}\right) \cdot \cos\left(\frac{6x-4x}{2}\right) \right] \cdot \left[ 2 \sin\left(\frac{6x-4x}{2}\right) \cdot \cos\left(\frac{6x+4x}{2}\right) \right]$

=  $4 \cdot \sin 5x \cdot \cos x \cdot \sin x \cdot \cos 5x$

=  $(2 \sin 5x \cdot \cos 5x) \cdot (2 \sin x \cdot \cos x)$

=  $(\sin 10x) \cdot (\sin 2x)$

= RHS.

Apply  $\begin{cases} \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) \\ \sin A - \sin B = 2 \sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right) \end{cases}$

$$(13) \quad \cos^2 2x - \cos^2 6x = \sin 4x \cdot \sin 8x$$

$$\text{LHS} = \cos^2 2x - \cos^2 6x \quad (a^2 - b^2)$$

$$= (\cos 2x + \cos 6x) (\cos 2x - \cos 6x)$$

$$= \left[ 2 \cos \left( \frac{2x+6x}{2} \right) \cdot \cos \left( \frac{2x-6x}{2} \right) \right] \cdot \left[ -2 \sin \left( \frac{2x+6x}{2} \right) \cdot \sin \left( \frac{2x-6x}{2} \right) \right]$$

$$= -4 \cos(4x) \cdot \cos(-2x) \cdot \sin(4x) \cdot \sin(-2x)$$

$$\left. \begin{array}{l} \cos(-\theta) = \cos \theta \\ \sin(-\theta) = -\sin \theta \end{array} \right\}$$

$$= 4 \cos 4x \cdot \cos 2x \cdot \sin 4x \cdot \sin 2x$$

$$= (2 \cos 4x \cdot \sin 4x) \cdot (2 \cos 2x \cdot \sin 2x)$$

$$= \sin 8x \cdot \sin 4x$$

$$= \text{RHS.}$$

$$\begin{aligned} \text{Apply } \cos A + \cos B &= 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2} \\ \cos A - \cos B &= -2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2} \end{aligned}$$

$$\textcircled{14} \quad \sin 2x + 2\sin 4x + \sin 6x = \underline{4\cos^2 x \cdot \sin 4x}$$

$$\text{LHS} = \sin 2x + 2\sin 4x + \sin 6x$$

$$= (\sin 2x + \sin 6x) + 2\sin 4x$$

$$\boxed{\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)}$$

$$= 2\sin(4x) \cdot \cos(-2x) \quad \left(\cos(-\theta) = \cos\theta\right)$$

$$= \underline{2\sin 4x \cdot \cos 2x} + \underline{2\sin 4x}$$

$$= \underline{2\sin 4x} \cdot \left\{ \underline{\cos 2x + 1} \right\}$$

$$\left( \because \cos 2\theta = 2\cos^2\theta - 1 \right)$$

$$= 2\sin 4x \cdot \left\{ \underline{2\cos^2 x - 1} + 1 \right\}$$

$$= 4\cos^2 x \cdot \sin 4x = \text{RHS. } \checkmark$$

# Toppers Village

Class - 11 - Maths x Exercise 3.3

(15)  $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

$\Rightarrow \frac{\cos 4x}{\sin 4x} \left( 2 \sin \left( \frac{5x+3x}{2} \right) \cdot \cos \left( \frac{5x-3x}{2} \right) \right) = \frac{\cos x}{\sin x} \left( 2 \sin \left( \frac{5x-3x}{2} \right) \cdot \cos \left( \frac{5x+3x}{2} \right) \right)$

$\Rightarrow \frac{\cos 4x}{\sin 4x} \left( 2 \cdot \cancel{\sin 4x} \cdot \cos x \right) = \frac{\cos x}{\cancel{\sin x}} \cdot \left( 2 \cancel{\sin x} \cdot \cos 4x \right)$

$\Rightarrow 2 \cdot \cos x \cdot \cos 4x = 2 \cos x \cdot \cos 4x$

LHS = RHS ✓

(16)  $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$

$\Rightarrow \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = \frac{-2 \cancel{\sin 7x} \cdot \sin 2x}{2 \cancel{\sin 7x} \cdot \cos 10x}$

$\Rightarrow \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x} = \underline{\underline{RHS}}$

APPLY  
 $\cos A - \cos B =$   
 $\sin A - \sin B =$

$$(17) \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

$$\text{LHS} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} \quad \left( \begin{array}{l} \text{Apply} \\ \sin A + \sin B = \equiv \\ \cos A + \cos B = \equiv \end{array} \right)$$

$$= \frac{2 \sin \left( \frac{5x+3x}{2} \right) \cdot \cos \left( \frac{5x-3x}{2} \right)}{2 \cos \left( \frac{5x+3x}{2} \right) \cdot \cos \left( \frac{5x-3x}{2} \right)}$$

$$= \frac{\cancel{2} \sin 4x \cdot \cancel{\cos x}}{\cancel{2} \cos 4x \cdot \cancel{\cos x}}$$

$$= \tan 4x$$

$$(18) \frac{\sin x - \sin y}{\cos x + \cos y} = \tan \left( \frac{x-y}{2} \right)$$

$$\text{LHS} = \frac{\sin x - \sin y}{\cos x + \cos y} \quad \left( \begin{array}{l} \text{Apply} \\ \sin A - \sin B = \\ \cos A + \cos B = \end{array} \right)$$

$$= \frac{\cancel{2} \sin \left( \frac{x-y}{2} \right) \cdot \cancel{\cos \left( \frac{x+y}{2} \right)}}{\cancel{2} \cos \left( \frac{x+y}{2} \right) \cdot \cos \left( \frac{x-y}{2} \right)}$$

$$= \frac{\sin \left( \frac{x-y}{2} \right)}{\cos \left( \frac{x-y}{2} \right)} = \tan \left( \frac{x-y}{2} \right) = \underline{\text{RHS}}$$

# Toppers Village

Class 11 Maths × Exercise 3.3

$$(19) \quad \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

$$\text{LHS} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{\cancel{2} \sin\left(\frac{x+3x}{2}\right) \cdot \cancel{\cos}\left(\frac{x-3x}{2}\right)}{\cancel{2} \cos\left(\frac{x+3x}{2}\right) \cdot \cancel{\cos}\left(\frac{x-3x}{2}\right)}$$

$$= \tan 2x = \underline{\text{RHS.}}$$

(20)

$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$$

$$\text{LHS} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} \rightarrow \begin{array}{l} \text{sin A - sin B} \\ \cos 2x = \cos^2 x - \sin^2 x \\ \textcircled{-} \cos 2x = -\cos^2 x + \sin^2 x \end{array}$$

$$= 2 \sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right)$$

$$= \frac{2 \sin\left(\frac{x-3x}{2}\right) \cdot \cos\left(\frac{x+3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2 \sin(-x) \cdot \cancel{\cos} 2x}{-\cos 2x} = \frac{-2 \sin x}{-1}$$

$$= 2 \sin x = \text{RHS.}$$

21 <sup>☆</sup>

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

$$\text{LHS} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2 \cos(3x) \cdot \cos x + \cos 3x}{2 \sin 3x \cdot \cos x + \sin 3x}$$

$$= \frac{\cos 3x [2 \cos x + 1]}{\sin 3x [2 \cos x + 1]} = \cot 3x = \text{RHS. } \checkmark$$

$$\begin{aligned} \cos(A) + \cos(B) &= 2 \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) \\ &= 2 \cos \frac{7x}{2} \cdot \cos \frac{x}{2} + \cos 2x \end{aligned}$$

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\begin{cases} \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cdot \cos \left( \frac{A-B}{2} \right) \\ \sin A - \sin B = 2 \sin \left( \frac{A-B}{2} \right) \cdot \cos \left( \frac{A+B}{2} \right) \\ \cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cdot \cos \left( \frac{A-B}{2} \right) \\ \star \cos A - \cos B = -2 \sin \left( \frac{A+B}{2} \right) \cdot \sin \left( \frac{A-B}{2} \right) \end{cases}$$

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 2\cos^2 \theta - 1 \quad \checkmark \\ &= 1 - 2\sin^2 \theta \\ &= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \end{aligned}$$

$$2 \sin \theta \cdot \cos \theta = \sin 2\theta$$

Formulae for Exercise 3.3.

Q 12 to Q 21

Toppers Village

Class-11-Maths    Exercise 3.3

Q.22

$$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$$

~~Cot x cot 2x~~

Proof:       $3x = (2x + x)$

By taking 'tan' both sides

$$\Rightarrow \tan 3x = \tan(2x + x)$$

$$\Rightarrow \tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x}$$

$$\textcircled{\tan} \rightarrow \textcircled{\frac{1}{\cot}}$$

$$\Rightarrow \frac{1}{\cot 3x} = \frac{\frac{1}{\cot 2x} + \frac{1}{\cot x}}{\frac{1}{1} - \frac{1}{\cot 2x} \cdot \frac{1}{\cot x}}$$

$$\Rightarrow \frac{1}{\cot 3x} = \frac{\frac{\cot x + \cot 2x}{\cot x \cdot \cot 2x}}{\frac{\cot x \cdot \cot 2x - 1}{\cot x \cdot \cot 2x}}$$

$$\Rightarrow \frac{1}{\cot 3x} = \frac{\cot x + \cot 2x}{\cot x \cdot \cot 2x - 1}$$

$$\Rightarrow \cot x \cdot \cot 2x - 1 = \cot 3x \cdot \cot x + \cot 3x \cdot \cot 2x$$

$$\Rightarrow \cot x \cdot \cot 2x - \cot 3x \cdot \cot x - \cot 3x \cdot \cot 2x = 1$$

H.P.

Q.23

$$\tan 4x = \frac{4 \tan x \cdot (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

LHS. =  $\tan 4x$

$$= \tan(2(2x))$$

$$= \frac{2 \tan(2x)}{1 - \tan^2(2x)}$$

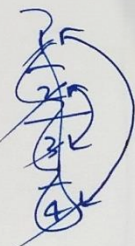
$$= 2 \left( \frac{2 \tan x}{1 - \tan^2 x} \right)$$

$$\frac{1}{1} - \left( \frac{2 \tan x}{1 - \tan^2 x} \right)^2$$

$$= \frac{(4 \tan x)}{(1 - \tan^2 x)} \cdot \frac{1}{(1 - \tan^2 x)^2 - (2 \tan x)^2}$$

$\tan 2\theta$   
 $\Downarrow$   
 $\frac{2 \tan \theta}{1 - \tan^2 \theta}$

$\tan 2x$   
 $= \frac{2 \tan x}{1 - \tan^2 x}$



$$= \frac{4 \tan x \cdot (1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x}$$
$$= \frac{4 \tan x \cdot (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

= RHS.

Q.22, Q.23

X  $\cot(x+y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$

✓  $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$

✓  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

# Toppers Village

## Class-11- maths. Exercise 3.3

Q.24

Prove that

$$\cos 4x = 1 - 8 \sin^2 x \cdot \cos^2 x$$

Proof:

$$\begin{aligned} \text{LHS} &= \cos 4x \\ &= 1 - 2 \sin^2(2x) \quad \left( \begin{array}{l} \because \cos 2\theta \\ = 1 - 2 \sin^2 \theta \end{array} \right) \\ &= 1 - 2 (\sin 2x)^2 \quad \left( \theta = 2x \right) \\ &= 1 - 2 (2 \sin x \cdot \cos x)^2 \quad \left( \begin{array}{l} \sin 2x = \\ 2 \sin x \cdot \cos x \end{array} \right) \\ &= 1 - 2 (4 \sin^2 x \cdot \cos^2 x) \\ &= 1 - 8 \sin^2 x \cdot \cos^2 x \\ &= \text{RHS.} \end{aligned}$$

$$\cos 2x = \begin{cases} \rightarrow \cos^2 x - \sin^2 x \\ \rightarrow 2 \cos^2 x - 1 \\ \rightarrow 1 - 2 \sin^2 x \\ \rightarrow \frac{1 - \tan^2 x}{1 + \tan^2 x} \end{cases}$$

Q24

Q25

$$\sin 2x = \begin{cases} \rightarrow 2 \sin x \cos x \\ \rightarrow \frac{2 \tan x}{1 + \tan^2 x} \end{cases}$$

$$\cos 3x = \begin{array}{l} 4 \cos^3 x - 3 \cos x \\ 4^3 - 3^1 \end{array}$$

Q.25 Prove that

$$\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

$$\text{LHS} = \cos 6x$$

$$= \cos 3(2x)$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$= 4 \cos^3(2x) - 3 \cos(2x)$$

$$= 4 (\cos 2x)^3 - 3 (\cos 2x)$$

$$= 4 (2 \cos^2 x - 1)^3 - 3 (2 \cos^2 x - 1)$$

$$\Rightarrow (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$= 4 [(2 \cos^2 x)^3 - 3 \cdot (2 \cos^2 x)^2 \cdot 1$$

$$+ 3(2 \cos^2 x) \cdot 1^2 - 1^3]$$

$$- 6 \cos^2 x + 3$$

$$= 4 (8 \cos^6 x - 12 \cos^4 x + 6 \cos^2 x - 1) - 6 \cos^2 x + 3$$

$$= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

# Toppers Village.

## Trigonometric Equations.

Before exercise **3.4**

# Algebraic equations

$$2x - 1 = 0 \rightarrow x = \frac{1}{2}$$

No. of Solutions  $\rightarrow$  Fixed.

$$x^2 - 5x + 6 = 0$$

$$\rightarrow x = 2, 3$$

# Trigonometric equations.

No. of Solutions  $\rightarrow \infty$

Unknown  
 $\downarrow$   
Angle

$$\sin x = \frac{1}{2}$$

Angle

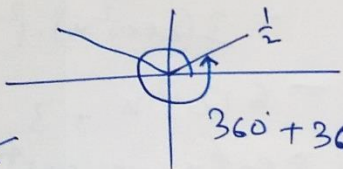
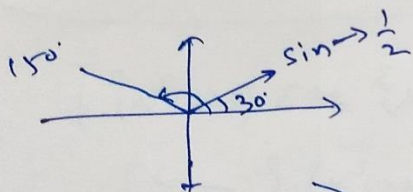
$$\theta = 30^\circ = \frac{\pi}{6}$$

$$\theta = 150^\circ = \frac{5\pi}{6}$$

$$\theta = 390^\circ = \frac{7\pi}{6}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\sin 150^\circ = \frac{1}{2}$$



$$360^\circ + 30^\circ = 390^\circ$$

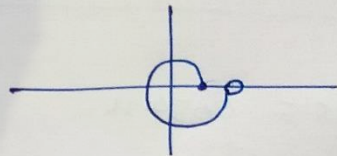
$$\theta = 750^\circ$$



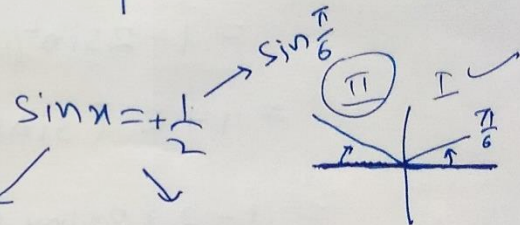
## Principal solution.

Angle  $\rightarrow [0, 2\pi)$

$$0 \leq \text{Angle} < 2\pi$$



e.g.



$$x = \frac{\pi}{6}$$

$$x = \frac{5\pi}{6} = \pi - \frac{\pi}{6}$$

(I)

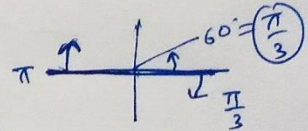
(II)

Pri. sol<sup>n</sup> =  $\frac{\pi}{6}, \frac{5\pi}{6} \in [0, 2\pi)$

$$\tan x = -\sqrt{3}$$

II  
 $\downarrow$   
 $\pi - \frac{\pi}{3}$

IV  
 $\rightarrow$   
 $2\pi - \frac{\pi}{3}$



$$= \frac{2\pi}{3}$$

$$= \frac{5\pi}{3}$$

# General Solutions of Trigonometric Equations.

$\sin x = \sin \theta$	$x = n\pi + (-1)^n \theta$	$n \in \mathbb{Z}$
$\operatorname{cosec} x = \operatorname{cosec} \theta$		
$\cos x = \cos \theta$	$x = 2n\pi \pm \theta$	
$\sec x = \sec \theta$		
$\tan x = \tan \theta$	$x = n\pi + \theta$	
$\cot x = \cot \theta$		

Proof: (I)  $\sin x = \sin \theta$

$$\Rightarrow \sin x - \sin \theta = 0$$

$$\Rightarrow 2 \sin\left(\frac{x-\theta}{2}\right) \cdot \cos\left(\frac{x+\theta}{2}\right) = 0$$

$$\sin\left(\frac{x-\theta}{2}\right) = 0$$

$$\Rightarrow \frac{x-\theta}{2} = n\pi$$

$$\Rightarrow \boxed{x = 2n\pi + \theta}$$

Even  $\oplus$

$$\cos\left(\frac{x+\theta}{2}\right) = 0$$

$$\frac{x+\theta}{2} = (2n+1)\frac{\pi}{2}$$

$$\boxed{x = (2n+1)\pi - \theta}$$

odd  $\ominus$

$(n-1)(n-2)=0$   
 $\downarrow$   
 $n=1$     $n=2$

$$x = 2n\pi + \theta$$

$$x = (2n+1)\pi - \theta$$

$$\boxed{x = n\pi + (-1)^n \theta}$$

$n = \text{even}$   
 $n = \text{odd}$

(II)  $\cos x = \cos \theta$

$$\Rightarrow \cos x - \cos \theta = 0$$

$$\Rightarrow -2 \sin\left(\frac{x+\theta}{2}\right) \cdot \sin\left(\frac{x-\theta}{2}\right) = 0$$

$$\sin\left(\frac{x+\theta}{2}\right) = 0$$

$$\frac{x+\theta}{2} = n\pi$$

$$\boxed{x = 2n\pi - \theta}$$

$$\boxed{x = 2n\pi + \theta}$$

$$\sin\left(\frac{x-\theta}{2}\right) = 0$$

$$\frac{x-\theta}{2} = n\pi$$

$$\boxed{x = 2n\pi + \theta}$$

$$\textcircled{\text{III}} \quad \tan x = \tan \theta$$

$$\Rightarrow \tan x - \tan \theta = 0$$

$$\Rightarrow \frac{\sin x}{\cos x} - \frac{\sin \theta}{\cos \theta} = 0$$

$$\Rightarrow \frac{\sin x \cdot \cos \theta - \cos x \cdot \sin \theta}{\cos x \cdot \cos \theta} = 0$$

$$\Rightarrow \underbrace{\sin x \cdot \cos \theta - \cos x \cdot \sin \theta}_{\sin(x-\theta)} = 0$$

$$\Rightarrow \sin(x-\theta) = 0$$

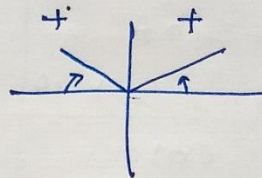
$$\Rightarrow x - \theta = n\pi$$

$$\boxed{x = n\pi + \theta}$$

Note:

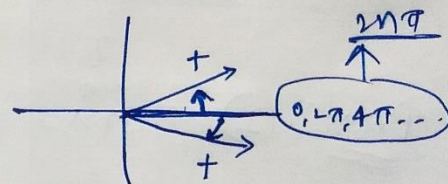
$$\sin x = \sin \theta$$

$$\boxed{x = n\pi + (-1)^n \theta}$$



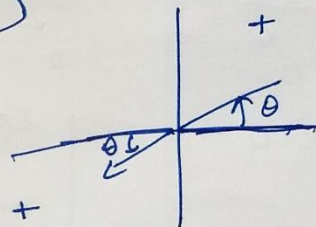
$$\cos x = \cos \theta$$

$$\boxed{x = 2n\pi \pm \theta}$$



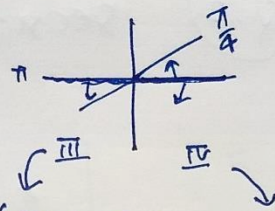
$$\tan x = \tan \theta$$

$$\boxed{x = n\pi + \theta}$$



## Example

General Sol<sup>n</sup>



$$\sin x = -\frac{1}{\sqrt{2}} = \sin\left(\pi + \frac{\pi}{4}\right) \quad \text{or} \quad \sin\left(2\pi - \frac{\pi}{4}\right)$$

$$\sin x = \sin\left(\frac{5\pi}{4}\right)$$

$$x = n\pi + (-1)^n \theta \quad n \in \mathbb{I}$$

General Sol<sup>n</sup>

$$x = n\pi + (-1)^n \cdot \frac{5\pi}{4}$$

Formula:

$$n = 0$$
$$x = \frac{5\pi}{4} \quad \checkmark$$

$$\sin x = \sin \frac{7\pi}{4}$$

$$x = n\pi + (-1)^n \cdot \frac{7\pi}{4} \quad n \in \mathbb{I}$$

Both are correct.

But look different

$$n = 3$$
$$x = \frac{3\pi}{1} - \frac{7\pi}{4} = \frac{12\pi - 7\pi}{4} = \frac{5\pi}{4}$$

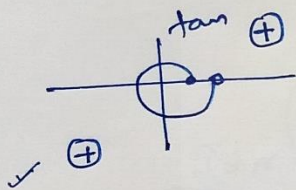
# Toppers Village

Class - 11 - Maths x Exercise 3.4

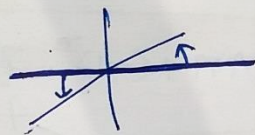
Principal & General Solutions

Trigonometric Equations

Q.1  $\tan x = \sqrt{3}$



$x = \frac{\pi}{3} = 60^\circ$



Principal sol<sup>n</sup>  $[0, 2\pi)$

$$\left. \begin{array}{l} \text{I} \rightarrow \frac{\pi}{3} = \frac{\pi}{3} \\ \text{III} \rightarrow \pi + \frac{\pi}{3} = \frac{4\pi}{3} \end{array} \right\}$$

General sol<sup>n</sup>

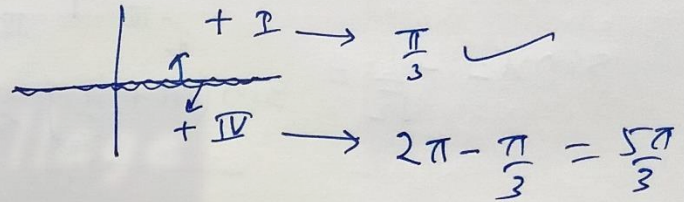
$\sqrt{3} = \tan x = \tan \frac{\pi}{3}$

$x = n\pi + \frac{\pi}{3}$

Q.2

$\sec x = 2 = \sec \frac{\pi}{3}$

$\cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}$



Principal sol<sup>n</sup>  $= \frac{\pi}{3}, \frac{5\pi}{3}$

General sol<sup>n</sup>

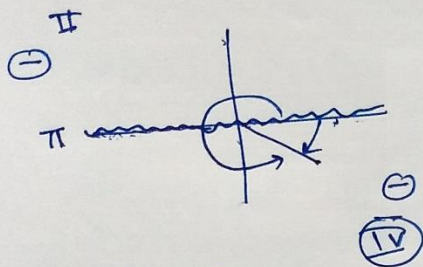
$x = 2n\pi \pm \theta \quad n \in \mathbb{I}$

$x = 2n\pi \pm \frac{\pi}{3}$

Q.3

$$\cot x = -\sqrt{3} = \cot \frac{5\pi}{6} = \cot \frac{11\pi}{6}$$

$$\left( 30^\circ, \frac{\pi}{6} \right)$$



Principal:  $\Rightarrow \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\rightarrow 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

General sol<sup>n</sup>.

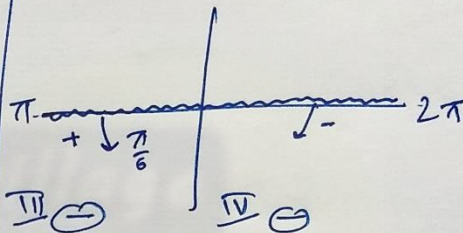
$$x = n\pi + \theta, n \in \mathbb{I}$$

$$x = n\pi + \frac{5\pi}{6}$$

Q.4

$$\operatorname{cosec} x = -2$$

$$\left( \frac{\pi}{6}, 30^\circ \right)$$



Principal sol<sup>n</sup>  $\Rightarrow \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\rightarrow 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

General,

$$x = n\pi + (-1)^n \cdot \theta$$

$$x = n\pi + (-1)^n \frac{7\pi}{6}$$

# Toppers Village

## Class - 11 - Maths x Exercise 3.4

Q.5

General solutions

$$\cos 4x = \cos 2x$$

$$\Rightarrow 4x = 2n\pi \pm 2x$$

$$\cos x = \cos \theta$$

↓

$$x = 2n\pi \pm \theta$$

$$n \in \mathbb{I}$$

⊕

$$4x = 2n\pi + 2x$$

$$\Rightarrow 4x - 2x = 2n\pi$$

$$\Rightarrow 2x = 2n\pi$$

$$\boxed{x = n\pi}, n \in \mathbb{I}$$

⊖

$$4x = 2n\pi - 2x$$

$$\Rightarrow 4x + 2x = 2n\pi$$

$$\Rightarrow 6x = 2n\pi$$

$$\Rightarrow \boxed{x = \frac{n\pi}{3}}, n \in \mathbb{I}$$

$$\textcircled{6} \quad \cos 3x + \cos x - \cos 2x = 0$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2}$$

$$\Rightarrow 2 \cos \frac{3x+x}{2} \cdot \cos \frac{3x-x}{2} - \cos 2x = 0$$

$$\Rightarrow 2 \cos 2x \cdot \cos x - \cos 2x = 0$$

$$\Rightarrow \cos 2x \cdot [2 \cos x - 1] = 0$$

$$\cos 2x = 0$$

$$\Rightarrow 2x = (2n+1) \frac{\pi}{2}$$

$$\Rightarrow \boxed{x = (2n+1) \frac{\pi}{2}}$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2} = \cos \frac{\pi}{3}$$

General sol<sup>n</sup>

$$\Rightarrow \boxed{x = 2n\pi \pm \frac{\pi}{3}}$$

$$\textcircled{7} \quad \sin 2x + \cos x = 0$$

$$\Rightarrow 2 \sin x \cdot \cos x + \cos x = 0$$

$$\Rightarrow \cos x \cdot [2 \sin x + 1] = 0$$

$$\cos x = 0$$

$$\boxed{x = (2n+1) \frac{\pi}{2}}$$

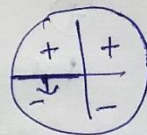
$$2 \sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$$\sin x = \sin \left( \frac{\pi + \pi}{6} \right)$$

$$\sin x = \sin \frac{7\pi}{6}$$

$$\Rightarrow \boxed{x = n\pi + (-1)^n \frac{7\pi}{6}}$$



# Toppers Village

Class - 11 - Maths × Exercise 3.4

**Q.8**  $\sec^2 2x = 1 + \tan^2 2x$

$\sec^2 \theta = 1 + \tan^2 \theta$

$\Rightarrow 1 + \tan^2 2x = 1 + \tan^2 2x$

$\Rightarrow \tan^2 2x + \tan^2 2x = 0$

$\Rightarrow \tan 2x \cdot [\tan 2x + 1] = 0$

$\tan 2x = 0$

$\Rightarrow 2x = n\pi$

$x = \frac{n\pi}{2}$

$\tan 2x = -1$  

⊖	+
+	⊖

$\tan 2x = \tan\left(\pi - \frac{\pi}{4}\right)$

$\tan 2x = \tan\left(\frac{3\pi}{4}\right)$

$2x = n\pi + \frac{3\pi}{4}$

$x = \frac{n\pi}{2} + \frac{3\pi}{8}$

**Q.9**  $\sin x + \sin 3x + \sin 5x = 0$

$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cdot \cos \frac{A-B}{2}$

$\Rightarrow \sin x + \sin 5x + \sin 3x = 0$

$\Rightarrow 2 \sin \frac{6x}{2} \cdot \cos \frac{4x}{2} + \sin 3x = 0$

$\Rightarrow 2 \sin 3x \cdot \cos 2x + \sin 3x = 0$

$\Rightarrow \sin 3x \cdot [2 \cos 2x + 1] = 0$

$3x = n\pi$

$x = \frac{n\pi}{3}$

$2 \cos 2x + 1 = 0$

$\Rightarrow \cos 2x = -\frac{1}{2} = \cos \frac{2\pi}{3}$

$2x = 2n\pi \pm \frac{2\pi}{3}$

$x = n\pi \pm \frac{\pi}{3}$

Toppers Village

Miscellaneous Exercise on Chapter 3

Trigonometric Functions

$$\textcircled{1} \quad 2 \cos \frac{\pi}{13} \cdot \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$$

LHS =

$$\Rightarrow 2 \cos \frac{\pi}{13} \cdot \cos \frac{9\pi}{13} + 2 \cos \left( \frac{3\pi}{13} + \frac{5\pi}{13} \right) \cdot \cos \left( \frac{5\pi}{13} - \frac{3\pi}{13} \right)$$

$$= 2 \cos \frac{\pi}{13} \cdot \cos \frac{9\pi}{13} + 2 \cos \left( \frac{4\pi}{13} \right) \cdot \cos \frac{\pi}{13}$$

$$= 2 \cos \frac{\pi}{13} \cdot \left[ \cos \frac{9\pi}{13} + \cos \frac{4\pi}{13} \right]$$

$$\frac{9\pi}{13} + \frac{4\pi}{13} = \pi$$

$$\left( \frac{9\pi}{13} \right) = \left( \pi - \frac{4\pi}{13} \right)$$

$$\Rightarrow \cos \left( \frac{9\pi}{13} \right) = \cos \left( \pi - \frac{4\pi}{13} \right)$$
$$\Rightarrow \cos \frac{9\pi}{13} = -\cos \frac{4\pi}{13}$$

✓ 16-13

Prove

$$= 2 \cos \frac{\pi}{13} \cdot \left\{ -\cos \frac{4\pi}{13} + \cos \frac{4\pi}{13} \right\}$$

$$= 0 = \text{RHS.}$$

$$\begin{cases} \cos A + \cos B = 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2} \\ \sin A + \sin B = 2 \sin \frac{A+B}{2} \cdot \cos \frac{A-B}{2} \\ \cos A - \cos B = -2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2} \\ \sin A - \sin B = 2 \sin \frac{A-B}{2} \cdot \cos \frac{A+B}{2} \end{cases}$$

+

OBSERVA

$$\textcircled{2} (\sin 3x + \sin x) \cdot \sin x + (\cos 3x - \cos x) \cdot \cos x = 0$$

Prove

$$\text{LHS} = (\sin 3x + \sin x) \cdot \sin x + (\cos 3x - \cos x) \cdot \cos x$$

$$= \left( 2 \sin \frac{3x+x}{2} \cdot \cos \frac{3x-x}{2} \right) \sin x + \left( -2 \sin \frac{3x+x}{2} \cdot \sin \frac{3x-x}{2} \right) \cdot \cos x$$

$$= \cancel{2 \sin 2x \cdot \cos x \cdot \sin x} - \cancel{2 \sin 2x \cdot \sin x \cdot \cos x}$$

$$= 0$$

# Toppers Village

class-11-maths

Misc. Ex. 3

Q.3

$$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \left( \frac{x+y}{2} \right)$$

↓

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2} \quad \checkmark$$

$$\sin A - \sin B = 2 \sin \frac{A-B}{2} \cdot \cos \frac{A+B}{2} \quad \checkmark$$

$$\text{LHS} = (\cos x + \cos y)^2 + (\sin x - \sin y)^2$$

$$= \left( 2 \cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2} \right)^2 + \left( 2 \sin \frac{x-y}{2} \cdot \cos \frac{x+y}{2} \right)^2$$

$$= \underline{4 \cos^2 \left( \frac{x+y}{2} \right) \cdot \cos^2 \left( \frac{x-y}{2} \right)} + \underline{4 \sin^2 \left( \frac{x-y}{2} \right) \cdot \cos^2 \left( \frac{x+y}{2} \right)}$$

$$= 4 \cos^2 \left( \frac{x+y}{2} \right) \cdot \left\{ \underbrace{\cos^2 \left( \frac{x-y}{2} \right) + \sin^2 \left( \frac{x-y}{2} \right)}_1 \right\}$$

$$= 4 \cos^2 \left( \frac{x+y}{2} \right) \cdot 1$$

$$= \text{RHS.}$$

$$\boxed{\text{Q.4}} \quad (\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2 \left( \frac{x-y}{2} \right)$$

$$\text{LHS} = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$$

$$\cos A - \cos B = \underline{\underline{-2 \sin \frac{A+B}{2} \cdot \sin \frac{A-B}{2}}}, \quad \sin A - \sin B = \underline{\underline{2 \sin \frac{A-B}{2} \cdot \cos \frac{A+B}{2}}}$$

$$= \left( -2 \sin \frac{x+y}{2} \cdot \sin \frac{x-y}{2} \right)^2 + \left( 2 \sin \frac{x-y}{2} \cdot \cos \frac{x+y}{2} \right)^2$$

$$= \underline{4 \sin^2 \left( \frac{x+y}{2} \right)} \cdot \underline{\sin^2 \left( \frac{x-y}{2} \right)} + \underline{4 \cdot \sin^2 \left( \frac{x-y}{2} \right)} \cdot \underline{\cos^2 \left( \frac{x+y}{2} \right)}$$

$$= 4 \sin^2 \left( \frac{x-y}{2} \right) \cdot \left\{ \sin^2 \left( \frac{x+y}{2} \right) + \cos^2 \left( \frac{x+y}{2} \right) \right\}$$

$$\Rightarrow 4 \sin^2 \left( \frac{x-y}{2} \right) = \text{RHS.} \quad \rightarrow |$$

# Toppers Village

## Miscellaneous Exercise on Chapter 3

**Q.5**

TO Prove

$$\begin{aligned} & \sin x + \sin 3x + \sin 5x + \sin 7x \\ &= 4 \cos x \cdot \cos 2x \cdot \sin 4x \end{aligned}$$

$$\text{LHS} = \sin x + \sin 3x + \sin 5x + \sin 7x$$

$$= 2 \sin \frac{4x}{2} \cdot \cos \frac{2x}{2} + 2 \sin \frac{12x}{2} \cdot \cos \frac{2x}{2}$$

$$= 2 \cos x \cdot \{ \sin 2x + \sin 6x \}$$

$$= 2 \cos x \cdot \{ 2 \sin 4x \cdot \cos 2x \}$$

$$= \text{RHS}$$

**Q.6**

$$\text{RHS} = \tan 6x$$

$$\text{LHS} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

$$= \frac{(\cancel{2} \sin 6x \cdot \cos x) + (\cancel{2} \sin 6x \cdot \cos 3x)}{(\cancel{2} \cos 6x \cdot \cos x) + (\cancel{2} \cos 6x \cdot \cos 3x)}$$

$$= \frac{\sin 6x \cdot (\cos x + \cos 3x)}{\cos 6x \cdot (\cos x + \cos 3x)}$$

$$= \tan 6x = \text{RHS}$$

Q.7

$$\sin 3x + \sin 2x - \sin x = 4 \sin x \cdot \cos \frac{x}{2} \cdot \cos \frac{3x}{2}$$

$$\text{LHS} = \underline{\sin 3x} + \sin 2x - \underline{\sin x}$$

$$= (\sin 3x - \sin x) + \sin 2x$$

$$= 2 \sin x \cdot \cos 2x + \underline{\sin 2x}$$

$$\sin 2x = 2 \sin x \cdot \cos x$$

$$= \underline{2 \sin x} \cdot \cos 2x + \underline{2 \sin x} \cdot \cos x$$

$$= 2 \sin x \cdot \{ \cos 2x + \cos x \}$$

$$= 2 \sin x \cdot \left\{ 2 \cos \frac{2x+x}{2} \cdot \cos \frac{2x-x}{2} \right\}$$

$$= \cancel{2 \sin x} \cdot \left\{ = 4 \left\{ \sin x \cdot \cos \frac{3x}{2} \cdot \cos \frac{x}{2} \right\} = \text{RHS.} \right.$$

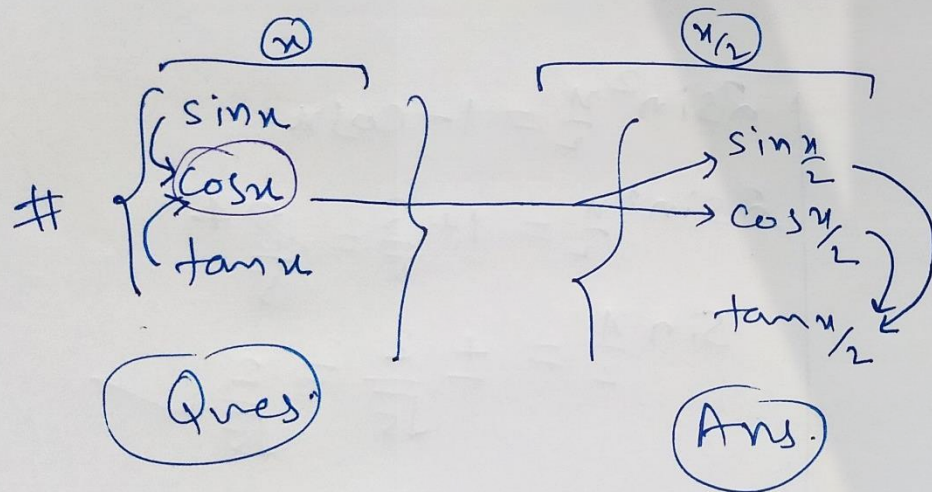
---

# Toppers Village

## Miscellaneous Exercise on Chapter (3)

Q8, Q9, Q10

$$\# \begin{cases} \cos 2\theta = 2\cos^2 \theta - 1 \rightarrow \cos x = 2\cos^2 \frac{x}{2} - 1 \rightarrow \boxed{2\cos^2 \frac{x}{2} = \cos x + 1} \\ \cos 2\theta = 1 - 2\sin^2 \theta \rightarrow \cos x = 1 - 2\sin^2 \frac{x}{2} \rightarrow \boxed{2\sin^2 \frac{x}{2} = 1 - \cos x} \end{cases}$$



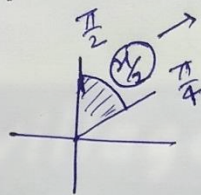
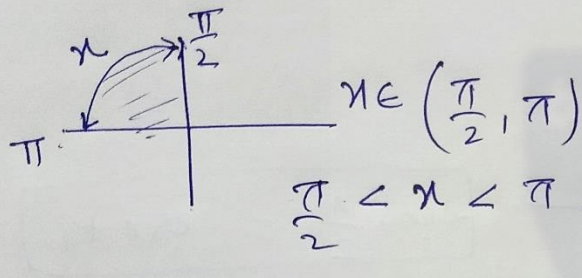
Q.8

$$\tan x = -\frac{4}{3}$$

II quadrant

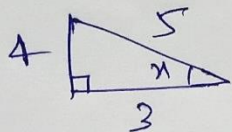
$$\left. \begin{array}{l} \sin \frac{x}{2} ? \\ \cos \frac{x}{2} ? \\ \tan \frac{x}{2} ? \end{array} \right\} \rightarrow \begin{array}{l} + \\ + \\ + \end{array}$$

I quadrant



$$\frac{\pi}{4} < \left(\frac{x}{2}\right) < \frac{\pi}{2} \quad \text{I}$$

$$\tan x = -\frac{4}{3} = \frac{P}{B}$$



$$\cos x = \frac{B}{H} = -\frac{3}{5}$$

II

$$2\sin^2 \frac{x}{2} = 1 - \cos x$$

$$\sin^2 \frac{x}{2} = 1 + \frac{3}{5} = \frac{8}{5}$$

$$\sin \frac{x}{2} = \pm \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \quad \checkmark$$

$$2\cos^2 \frac{x}{2} = 1 + \cos x$$

$$\Rightarrow 2\left(\cos \frac{x}{2}\right)^2 = 1 + \left(-\frac{3}{5}\right)$$

$$\Rightarrow \cancel{2} \left(\cos \frac{x}{2}\right)^2 = \frac{2}{5} \Rightarrow \cos \frac{x}{2} = \pm \frac{1}{\sqrt{5}} = +\frac{1}{\sqrt{5}} \quad \checkmark$$

$$\begin{array}{l} \cos \frac{x}{2} = \frac{1}{\sqrt{5}} \quad \checkmark \\ \sin \frac{x}{2} = \frac{2}{\sqrt{5}} \quad \checkmark \\ \tan \frac{x}{2} = 2 \quad \checkmark \end{array}$$



Q10

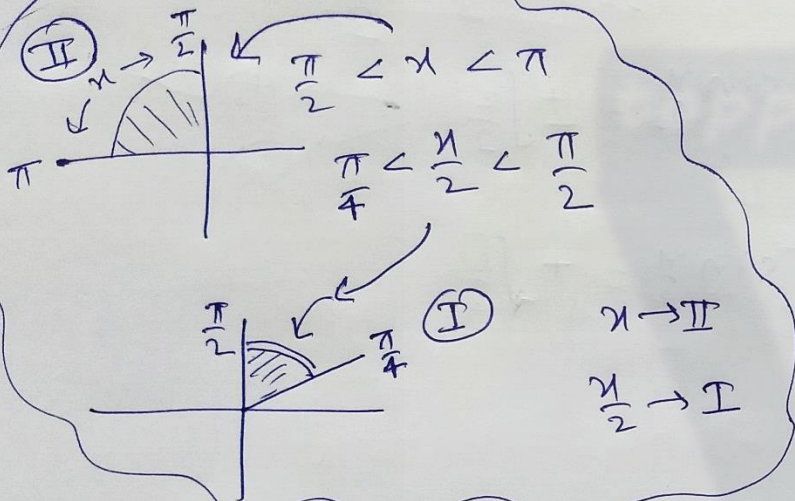
$$\sin x = \frac{1}{4}$$

II - Quadrant

$$\sin \frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}}$$

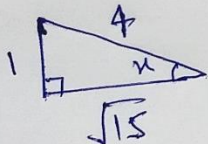
$$\cos \frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}}$$

$$\tan \frac{x}{2} = 4 + \sqrt{15}$$



$$\frac{P}{H} = \sin x = \frac{1}{4}$$

$x \rightarrow \text{II}$



$$\cos x = -\frac{\sqrt{15}}{4}$$

$$4 + \sqrt{15} = \sqrt{\frac{(4 + \sqrt{15})^2}{16 - 15}}$$

$$2 \cos^2 \frac{x}{2} = 1 + \cos x = 1 + \left(-\frac{\sqrt{15}}{4}\right)$$

$$\cos^2 \frac{x}{2} = \frac{4 - \sqrt{15}}{8}$$

$$\cos \frac{x}{2} = +\sqrt{\frac{4 - \sqrt{15}}{8}}$$

I-quad.

$$2 \sin^2 \frac{x}{2} = 1 - \cos x = 1 + \frac{\sqrt{15}}{4}$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{4 + \sqrt{15}}{4}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{4 + \sqrt{15}}{8}$$

$$\sin \frac{x}{2} = +\sqrt{\frac{4 + \sqrt{15}}{8}}$$

$\text{I}$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\sqrt{\frac{4 + \sqrt{15}}{8}}}{\sqrt{\frac{4 - \sqrt{15}}{8}}} = \frac{4 + \sqrt{15}}{4 - \sqrt{15}} \times \frac{4 + \sqrt{15}}{4 + \sqrt{15}}$$

## **All Useful Links:**

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** [www.toppersvillage.com](http://www.toppersvillage.com)
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** [www.facebook.com/ToppersVillage/](http://www.facebook.com/ToppersVillage/)
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>