

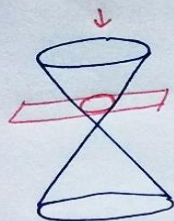


Class 11 Maths || Chapter 11 : Conic Sections || Class Notes

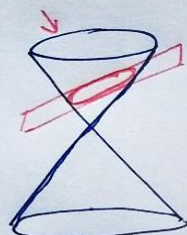
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Chapter - 11

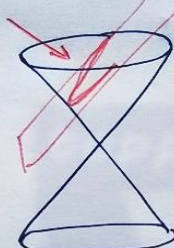
Conic Sections



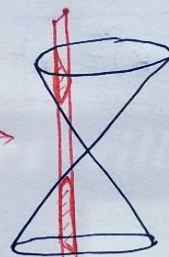
Circle



Ellipse



Parabola



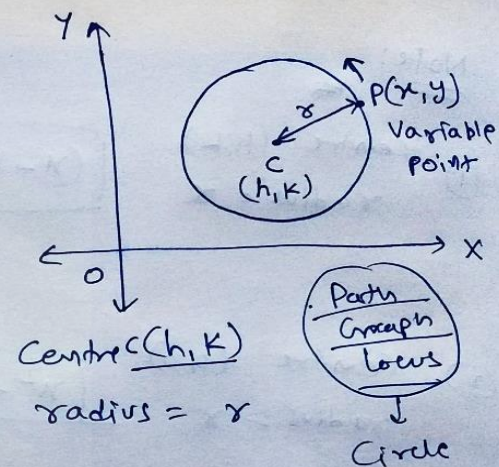
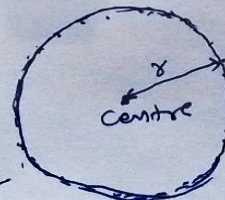
Hyperbola



Circle: A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

Fixed Point $\rightarrow$ Centre ✓

Fixed Distance $\rightarrow$ radius ✓



$$\therefore CP = r$$

$$\Rightarrow CP^2 = r^2$$

$$\Rightarrow (x-h)^2 + (y-k)^2 = r^2$$

Standard Form of a Circle

E.g. Centre (2, 3)
 $r = 5$
 $(x-2)^2 + (y-3)^2 = 5^2$

YouTube पर इस Chapter के FREE Videos देखिए (Playlist की Link नीचे है)

https://www.youtube.com/watch?v=azJ05Fmo0uU&list=PLPFRn0ppwwkRc_ztmMOiGjaTaBsDux1De

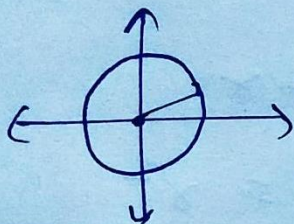
Note:

- ① Centre (h, k)
radius $= r$

$$\boxed{(x-h)^2 + (y-k)^2 = r^2} \quad \star$$

- ② Centre $(0, 0)$
radius $= r$

$$\boxed{x^2 + y^2 = r^2}$$



Exercise 11.1 - Circle

Find equation of circle.

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

Q.1 Centre $(0, 2)$

$$r = 2$$

Circle

$$(x-0)^2 + (y-2)^2 = 2^2$$

$$\Rightarrow x^2 + y^2 + \cancel{4} - 4y = \cancel{4}$$

$$\Rightarrow \boxed{x^2 + y^2 - 4y = 0}$$

Q.2 Centre $(-2, 3)$

$$r = 4$$

Circle

$$(x+2)^2 + (y-3)^2 = 16$$

$$\Rightarrow x^2 + 4 + 4x + y^2 + 9 - 6y = 16$$

$$\Rightarrow \boxed{x^2 + y^2 + 4x - 6y - 3 = 0}$$

Q.3 Centre $(\frac{1}{2}, \frac{1}{4})$

$$r = \frac{1}{12}$$

Circle

$$(x - \frac{1}{2})^2 + (y - \frac{1}{4})^2 = (\frac{1}{12})^2$$

$$\Rightarrow x^2 + \frac{1}{4} - x + y^2 + \frac{1}{16} - \frac{y}{2} = \frac{1}{144}$$

$$\Rightarrow 144x^2 + 144y^2 - 144x - 72y + 36 + 9 = 0$$

$$\Rightarrow \boxed{144x^2 + 144y^2 - 144x - 72y + 45 = 0}$$

Q.4 $r = \sqrt{2}$, Centre $(1, 1)$

Circle $(x-1)^2 + (y-1)^2 = (\sqrt{2})^2$

$$\Rightarrow x^2 - 2x + \cancel{1} + y^2 - 2y + \cancel{1} = \cancel{2}$$

$$\Rightarrow \boxed{x^2 + y^2 - 2x - 2y = 0}$$

Q.5

Centre $(-a, -b)$

$$r = \sqrt{a^2 - b^2}$$

Circle $(x+a)^2 + (y+b)^2 = (\sqrt{a^2 - b^2})^2$

$$\Rightarrow x^2 + a^2 + 2ax + y^2 + b^2 + 2by = a^2 - b^2$$

$$\Rightarrow \boxed{x^2 + y^2 + 2ax + 2by + 2b^2 = 0}$$

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Class - 11 - Maths

Conic Sections - Circle

Exercise (11-1)

✓ Centre (h, k)

✓ $r =$

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

Complete
Square
in x

$=$
in y

$$\boxed{Q.6} \quad (x+5)^2 + (y-3)^2 = 36 = 6^2$$

Centre = $(-5, 3)$

$$r = 6$$

$$\boxed{Q.7} \quad x^2 + y^2 - 4x - 8y - 45 = 0$$

we have to apply "Completing
the square method"

$$\underline{x^2 + y^2 - 4x - 8y - 45 = 0} \rightarrow$$

$$\Rightarrow \underline{x^2 - 4x} + \underline{y^2 - 8y} = 45$$

$$\Rightarrow \underline{x^2 - 4x + 4} + \underline{y^2 - 8y + 16} = 45 + 4 + 16$$

$$\Rightarrow (x-2)^2 + (y-4)^2 = 65$$

$$\Rightarrow (x-2)^2 + (y-4)^2 = (\sqrt{65})^2$$

Centre $(2, 4)$ ✓

$$r = \sqrt{65} \quad \checkmark$$

$$\boxed{Q.8} \quad \underline{x^2 + y^2 - 8x + 10y - 12 = 0} \rightarrow$$

$$\Rightarrow \underline{x^2 - 8x} + \underline{y^2 + 10y} = 12$$

$$\Rightarrow \underline{x^2 - 8x + 16} + \underline{y^2 + 10y + 25} = 12 + 16 + 25$$

$$\Rightarrow (x-4)^2 + (y+5)^2 = 53 = (\sqrt{53})^2$$

$$\boxed{\text{Centre } (4, -5) \quad r = \sqrt{53}}$$

Q.9 $2x^2 + 2y^2 - x = 0$

$$\Rightarrow (2x^2 - x + 2y^2 = 0) \div 2$$

$$\Rightarrow x^2 - \frac{x}{2} + \frac{1}{16} + y^2 = 0 + \frac{1}{16}$$

$$\Rightarrow \left(x - \frac{1}{4}\right)^2 + y^2 = \frac{1}{16} = \left(\frac{1}{4}\right)^2$$

Centre $\left(\frac{1}{4}, 0\right)$ ✓

$$r = \frac{1}{4} \checkmark$$

Q.10 ✓ Passing through $(4, 1)$ & $(6, 5)$

✓ Centre on $4x + y = 16$ ✓

Let the centre be (h, k) ✓

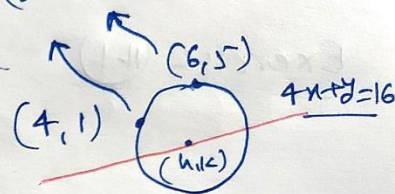
radius = r ✓

$$4h + k = 16 \quad \text{--- (1)}$$

Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

~~$$(x-h)^2 + (y-k)^2 = r^2$$~~



$$(4-h)^2 + (1-k)^2 = r^2 \quad \text{--- (2)}$$

$$(6-h)^2 + (5-k)^2 = r^2 \quad \text{--- (3)}$$

$$\rightarrow h^2 - 8h + k^2 - 2k + 17 = r^2$$

$$\rightarrow h^2 - 12h + k^2 - 10k + 61 = r^2$$

$$\begin{array}{ccccccc} - & + & - & + & - & + & - \end{array}$$

$$4h + 8k - 44 = 0$$

$$\Rightarrow h + 2k = 11 \quad \text{--- (4)}$$

$$h = 11 - 2k$$

By eqn (1) & (4):

$$\Rightarrow 4(11 - 2k) + k = 16$$

$$\Rightarrow 4(11-2K) + K = 16$$

$$\Rightarrow 44 - 8K + K = 16$$

$$\Rightarrow 28 = 7K$$

$$\Rightarrow \boxed{K = 4}$$

$$h = 11 - 2K$$

$$h = 11 - 2 \times 4 = 3$$

$$\text{Centre } (h, K) \equiv (3, 4)$$

$$\text{By eqn (2): } (4-h)^2 + (1-K)^2 = r^2$$

$$\Rightarrow (4-3)^2 + (1-4)^2 = r^2$$

$$\Rightarrow 1 + 9 = r^2$$

$$\Rightarrow r^2 = 10$$

$$r = \sqrt{10}$$

$$\underline{\text{Circle:}} \quad (x-3)^2 + (y-4)^2 = 10$$

Q.11 Passing through $(2, 3)$, $(-1, 1)$

✓ Centre on $x - 3y - 11 = 0$

Let centre (h, K) $\nearrow h - 3K - 11 = 0$ — (1)

radius = r

$$\underline{\text{Circle:}} \quad (x-h)^2 + (y-K)^2 = r^2$$

$$(2, 3) \rightarrow (2-h)^2 + (3-K)^2 = r^2 \text{ — (2)}$$

$$(-1, 1) \rightarrow (-1-h)^2 + (1-K)^2 = r^2 \text{ — (3)}$$

$$\boxed{11 = 6h + 4K} \text{ — (4)}$$

$$\text{By eqn (1) \& (4): } (h, K) \equiv \left(\frac{7}{2}, -\frac{5}{2}\right)$$

$$\text{By eqn (2) } r = \sqrt{\frac{65}{2}}$$

$$\underline{\text{Circle:}} \quad \left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{65}{2}$$

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Class-11-Maths. Exercise (11.1) Circle.

Centre (h, k)

radius $= r$

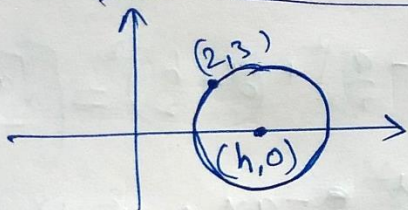
$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

Q.12

$r = 5$ ✓

Centre on x-axis $\rightarrow$ Centre $(h, 0)$

Passing through $(2, 3)$



~~Let~~ Circle $(x-h)^2 + y^2 = r^2 = 5^2$

$$\Rightarrow \boxed{x^2 - 2hx + h^2 + y^2 = 25}$$

$(2, 3)$ $\nearrow$ satisfy

$$\Rightarrow 4 - 4h + h^2 + 9 = 25$$

$$\Rightarrow h^2 - 4h - 12 = 0$$

$$\Rightarrow \underline{h^2 - 6h + 2h - 12} = 0$$

$$\Rightarrow h(h-6) + 2(h-6) = 0$$

$$\Rightarrow (h-6)(h+2) = 0$$

$\downarrow$

$$h = 6$$

Centre $(6, 0)$ ✓

$$r = 5$$

$\downarrow$

$$\boxed{(x-6)^2 + (y-0)^2 = 25}$$

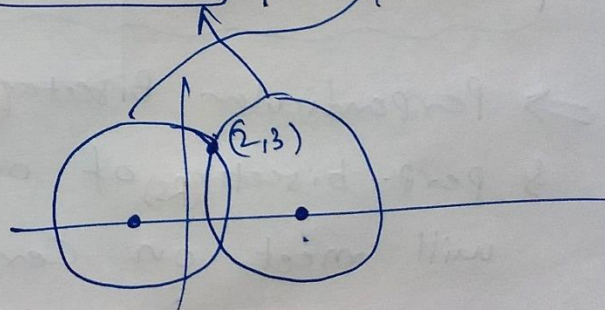
$\downarrow$

$$h = -2$$

Center $(-2, 0)$

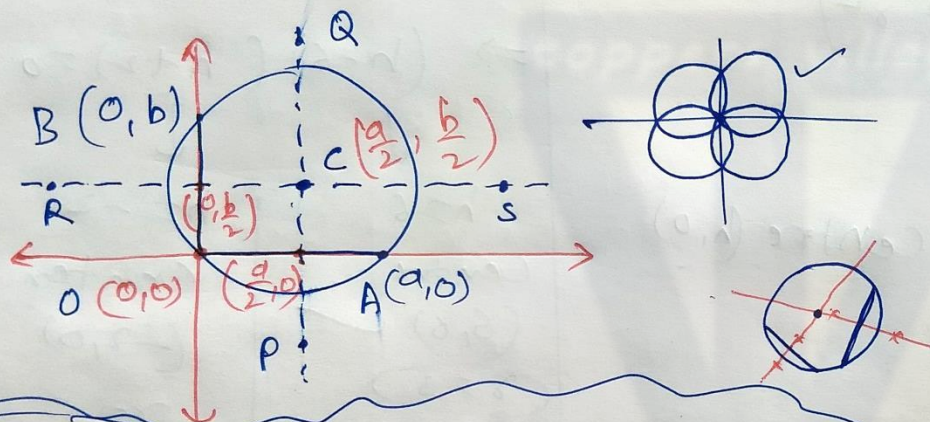
$$r = 5$$

$$\boxed{(x+2)^2 + y^2 = 25}$$



Q.13 ✓ Circle passing through $(0,0)$ ✓
 ✓ making intercepts a & b on axes.

↙ ↘
X-intercept Y-intercept



Class-9 → Perpendicular Bisector
 of any chord of a circle
 always passes through the 'centre'.

⇒ Perpendicular Bisector (PQ) of OA (chord)
 & perp. bisector (RS) of OB (chord)
 will meet on Centre 'C' $(\frac{a}{2}, \frac{b}{2})$

Centre • C $(\frac{a}{2}, \frac{b}{2})$

radius = $r = OC$

$$r = \sqrt{(\frac{a}{2} - 0)^2 + (\frac{b}{2} - 0)^2}$$

$$r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

$$r = \frac{\sqrt{a^2 + b^2}}{2}$$

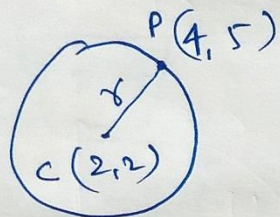
Circle

$$(x - \frac{a}{2})^2 + (y - \frac{b}{2})^2 = \left[\frac{\sqrt{a^2 + b^2}}{2} \right]^2$$

$$\Rightarrow x^2 + \frac{a^2}{4} - ax + y^2 + \frac{b^2}{4} - by = \frac{a^2 + b^2}{4}$$

$$\Rightarrow x^2 + y^2 - ax - by = 0$$

Q.14 - Centre (2,2)
 ✓ passing through (4,5)



$$r = CP = \sqrt{(4-2)^2 + (5-2)^2}$$

$$r = \sqrt{4 + 9}$$

$$r = \sqrt{13}, \text{ centre } (2,2)$$

Circle, $(x-2)^2 + (y-2)^2 = (\sqrt{13})^2$

$$\Rightarrow \underline{x^2} - \underline{4x} + 4 + \underline{y^2} - \underline{4y} + 4 = 13$$

$$\Rightarrow \boxed{x^2 + y^2 - 4x - 4y - 5 = 0}$$

Q.15

Circle $\boxed{x^2 + y^2 = 25}$

Position of point P(-2.5, 3.5) = ?



outside $P_2C > r$

on the circle $P_2C = r$

inside $P_1C < r$

$$(x-0)^2 + (y-0)^2 = 5^2$$

★ centre (0,0) C

★ $r = 5$

Distance b/w point P & centre C
 $\downarrow \qquad \qquad \qquad \downarrow$
 $(-2.5, 3.5) \qquad (0,0)$

$$CP = \sqrt{(-2.5)^2 + (3.5)^2}$$

$$CP = \sqrt{6.25 + 12.25}$$

$$CP = \sqrt{18.50}$$

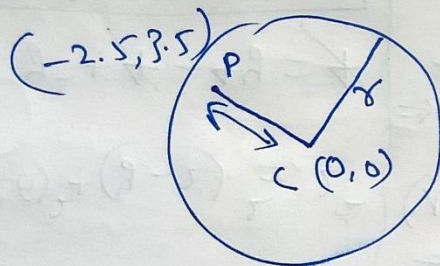
$$CP = \sqrt{18.50}$$

$$r = 5$$

$$\Rightarrow \boxed{CP < r}$$

$$\begin{array}{ccc} 16 & < 18.5 & < 25 \\ \downarrow & & \downarrow \\ 4 & \longleftrightarrow & 5 \end{array}$$

$$\begin{array}{ccc} \textcircled{CP^2} & < & \textcircled{r^2} \\ \downarrow & & \downarrow \\ \textcircled{18.5} & < & \textcircled{25} \end{array}$$

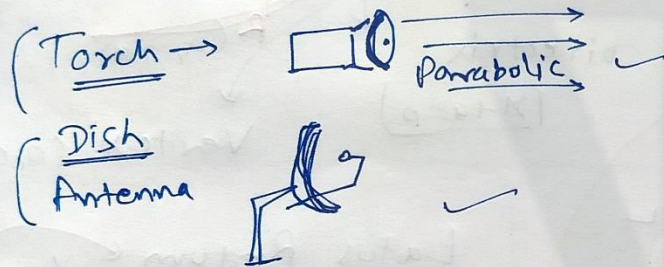


$P(-2.5, 3.5)$
 $\boxed{\text{inside the circle}}$

PARABOLA

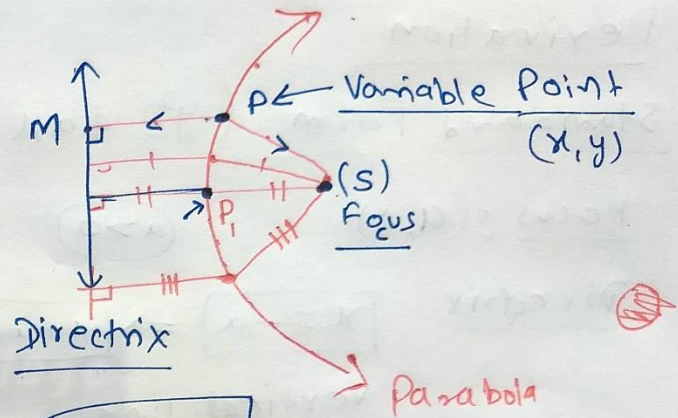
for Exercise (11.2)

Para bola
for throwing



Definition: set of all points
on a plane that are equidistant
from a fixed point (Focus)
& from a fixed line (Directrix).

Axis
Vertex

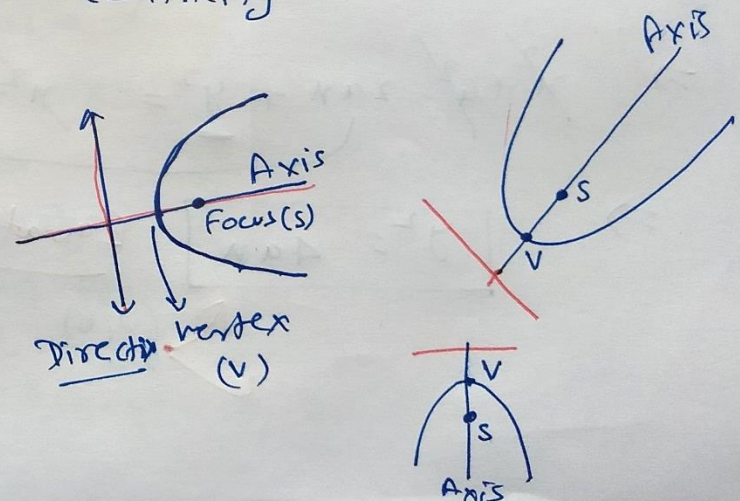


$$SP = PM$$

Parabola for

$$e = \frac{SP}{PM} = 1$$

eccentricity



Derivation

Standard Form ($y^2 = 4ax$)

Focus $S(a, 0)$

$(a > 0)$

Directrix

$$x = -a \Rightarrow x + a = 0$$

vertical line.

$$SP = PM$$

$$\Rightarrow \left(\sqrt{(x-a)^2 + (y-0)^2} \right)^2 = \left(\left| \frac{x+a}{\sqrt{1^2+0^2}} \right| \right)^2$$

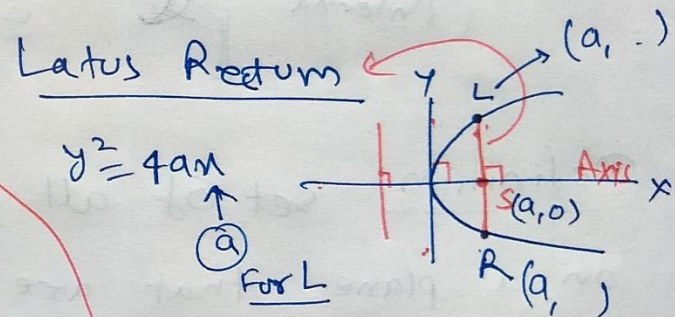
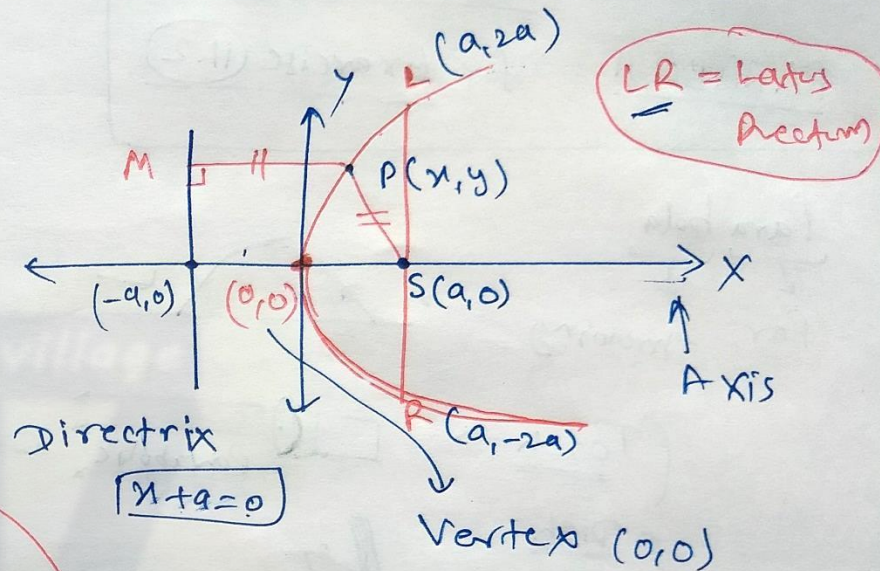
$$\Rightarrow x^2 + a^2 - 2ax + y^2 = x^2 + a^2 + 2ax$$

$\Rightarrow$

$$y^2 = 4ax$$

Equation of parabola

$(0, 0)$



$$y^2 = 4a \cdot a$$

$$y = \pm 2a \rightarrow y = +2a$$
$$\rightarrow y = -2a$$

Length of ~~the~~ latus Rectum = $4a$

I

$y^2 = 4ax$ ($a > 0$)

Position $\Rightarrow$ Rightward

Vertex $\Rightarrow (0, 0)$

Focus $\Rightarrow (a, 0)$

Directrix $\Rightarrow x = -a$

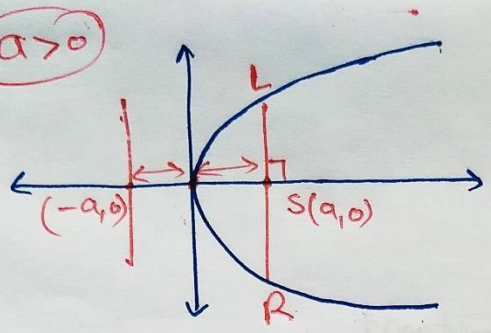
Axis $\Rightarrow x$ -axis

Symmetric About $\Rightarrow x$ -axis

Latus Rectum (LR)

$\rightarrow$ Equation $\Rightarrow x = a$

$\rightarrow$ Length $\Rightarrow 4a$



II

$y^2 = -4ax$

Position $\Rightarrow$ Leftward

Vertex $\Rightarrow (0, 0)$

Focus $\Rightarrow S(-a, 0)$

Directrix $\Rightarrow x = a$

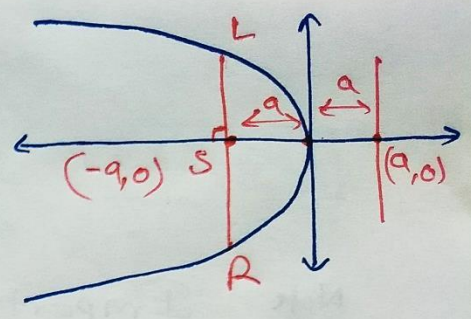
Axis $\Rightarrow x$ -axis

Symmetric About $\Rightarrow x$ -axis

Latus Rectum

$\rightarrow$ Equation $\Rightarrow x = -a$

$\rightarrow$ length $\Rightarrow 4a$



III

$x^2 = 4ay$

Position $\Rightarrow$ Upward

Vertex $\Rightarrow (0, 0)$

Focus $\Rightarrow S(0, a)$

Directrix $\Rightarrow y = -a$

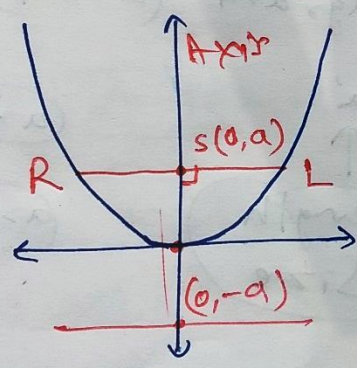
Axis $\Rightarrow y$ -axis

Sym. about $\Rightarrow y$ -axis

Latus Rectum

$\rightarrow$ Equation $\Rightarrow y = a$

$\rightarrow$ length $\Rightarrow 4a$



IV

$x^2 = -4ay$

Position $\Rightarrow$ Downward

Vertex $\Rightarrow (0, 0)$

Focus $\Rightarrow S(0, -a)$

Directrix $\Rightarrow y = a$

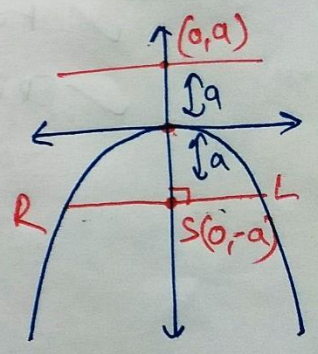
Axis $\Rightarrow y$ -axis

Sym. about $\Rightarrow y$ -axis

Latus Rectum (LR)

$\rightarrow$ Equation $\Rightarrow y = -a$

$\rightarrow$ length $\Rightarrow 4a$



Note

Important
things for
Derivation



✓ Focus

✓ Direction

Important
things for

Question Solving



★ { ① Diagram (Exact)
② 'a'



length
↖
Size

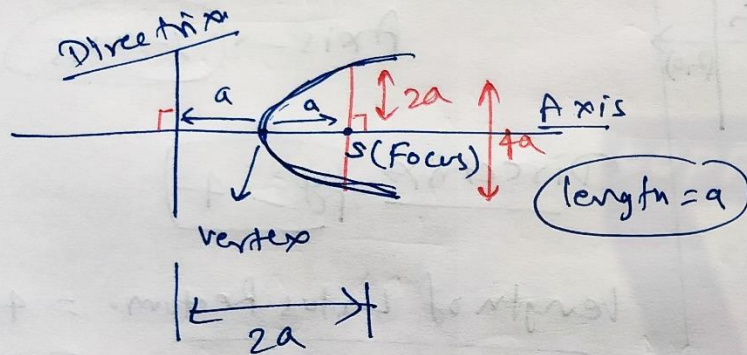
$a \leq$



$a \geq$



PARABOLA



Q.2

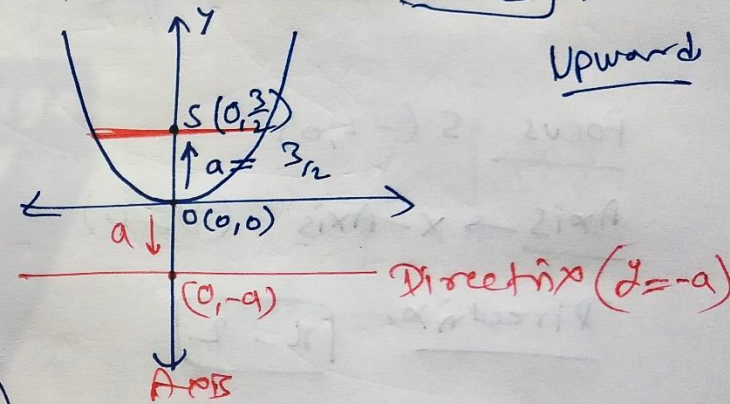
$$x^2 = 6y$$

$$\rightarrow x^2 = 4ay$$

$$6 = 4a$$

$$\Rightarrow a = \frac{3}{2}$$

Upward



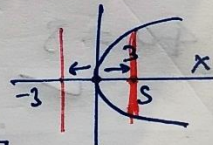
Q.1

$$y^2 = 12x$$

$$\Rightarrow y^2 = 4ax$$

$$4a = 12$$

$$a = 3$$



Coordinate of focus $\Rightarrow S(3, 0)$

Axis of Parabola $\Rightarrow$ x-axis $y = 0$

Eq. of Directrix $\Rightarrow x = -3$

length of latus Rectum $\Rightarrow 4a = 4 \times 3 = 12$

Focus = S(0, 3/2)

Axis $\rightarrow$ y-axis ($x = 0$)

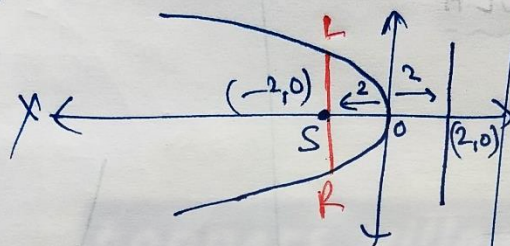
Directrix $\rightarrow y = -a$

length of Latus Rectum $\rightarrow 4a = 6$

Q.3 $y^2 = -8x \leftrightarrow y^2 = -4ax$

$-8 = -4a$

$a = 2$



Focus $S(-2,0)$

Axis $\rightarrow$ x-Axis ($y=0$)

Directrix $x = 2$

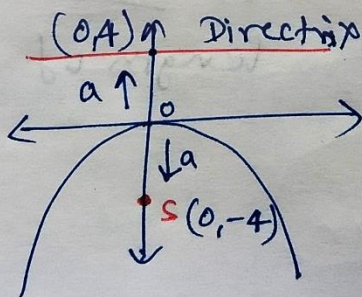
length of Latus Rectum (LR) $= 4a = 8$

Q.4 $x^2 = -16y \leftrightarrow x^2 = -4ay$

$-16 = -4a$

$4a = 16$

$a = 4$



Continue Q.4

Answer

focus $S(0,-4)$

Axis $\rightarrow$ y-axis

Directrix $y = 4$

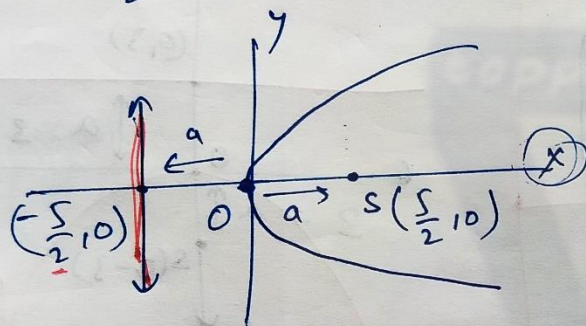
length of Latus Rectum $= 4a$
 $= 16$

Q.5

$$y^2 = 10x \leftrightarrow y^2 = 4ax$$

$$10 = 4a$$

$$\Rightarrow a = \frac{5}{2}$$



$$\text{Focus} = S\left(\frac{5}{2}, 0\right)$$

Axis: x-axis ($y=0$)

Directrix: $x = -\frac{5}{2}$

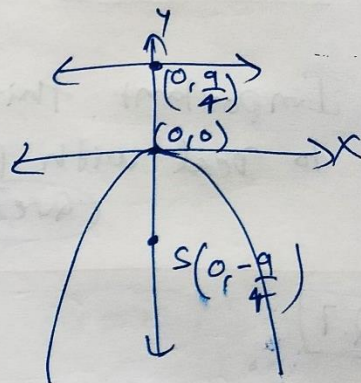
$$\begin{aligned} \text{Length of Latus Rectum} &= 4a \\ &= 10 \\ &\checkmark \end{aligned}$$

Q.6

$$x^2 = -9y \leftrightarrow x^2 = -4ay$$

$$9 = 4a$$

$$a = \frac{9}{4}$$



$$\text{Focus} = S\left(0, -\frac{9}{4}\right)$$

Axis $\rightarrow$ y-axis ($x=0$)

Directrix $\rightarrow y = \frac{9}{4}$

$$\begin{aligned} \text{Length of Latus Rectum} &= 4a \\ &= 9 \text{ units.} \end{aligned}$$

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Exercise 11.2 Parabola

★ Important things to Deal with parabola → ① Diagram (Exact) ② a

Question

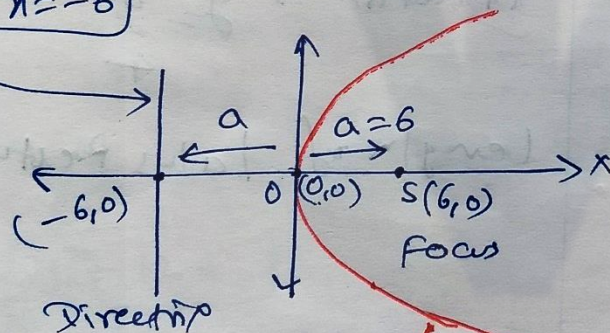
$y^2 = 4ax$	$x^2 = 4ay$
$y^2 = -4ax$	$x^2 = -4ay$

[Q.7]

Focus $S(6,0)$ → on $\oplus$ x-axis

Directrix $x = -6$

vertical



$a = 6$

Parabola

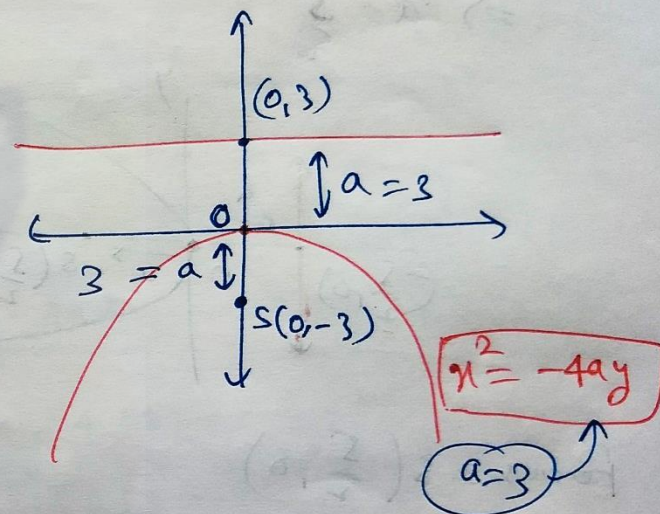
$$y^2 = 4ax$$

$$y^2 = 4 \times 6 \cdot x$$

$$y^2 = 24x$$

⑧ Focus $(0,-3)$

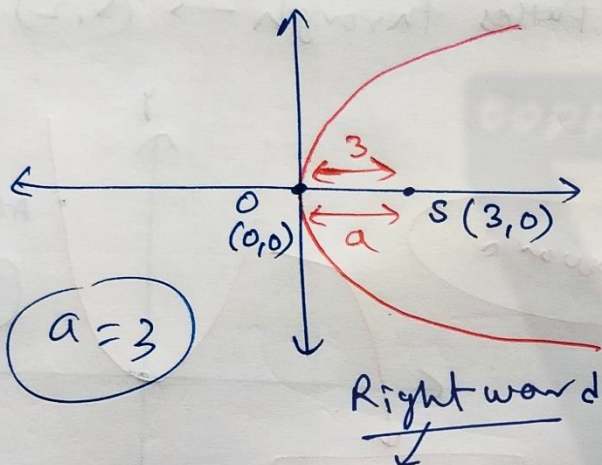
Directrix $y = 3$ → Horizontal



Parabola → $x^2 = -4(3) \cdot y$

$$x^2 = -12y$$

Q.9 vertex $(0,0)$
Focus $S(3,0)$



$a=3$

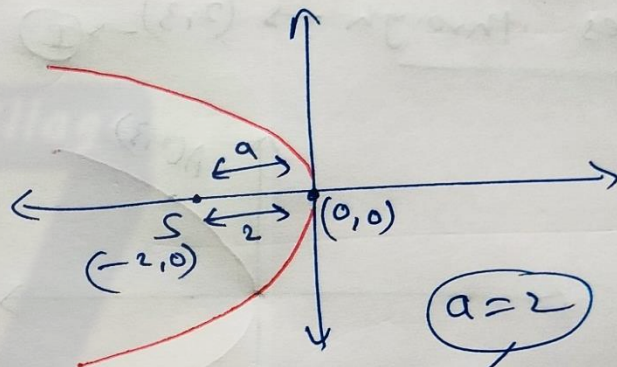
$y^2 = 4ax$
 $\Rightarrow a=3$

Parabola,

$y^2 = 4(3)(x)$

$y^2 = 12x$

Q.10 Vertex $(0,0)$
Focus $S(-2,0)$



$a=2$

$y^2 = -4ax$

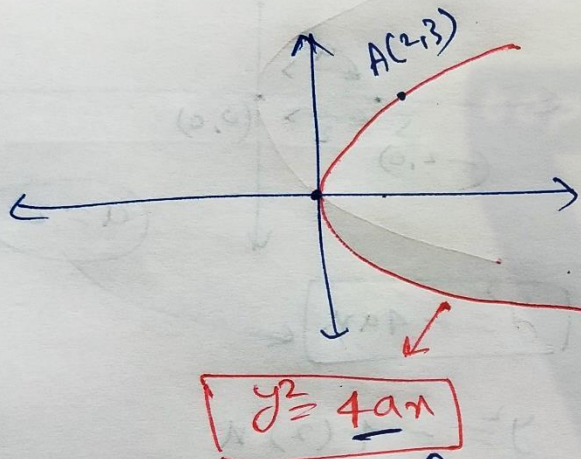
$y^2 = -4(2)x$

$y^2 = -8x$

[Q.11] Vertex $(0,0)$

Axis $\rightarrow$ x-axis 

Passes through $\rightarrow (2,3) \rightarrow \textcircled{I}$



$(2,3)$ will satisfy $\uparrow$

$$\Rightarrow (3)^2 = 4a \cdot (2)$$

$$\Rightarrow 9 = 8a$$

$$\Rightarrow \boxed{a = \frac{9}{8}}$$

Parabola
 $y^2 = 4ax$

$$\Rightarrow y^2 = 4\left(\frac{9}{8}\right)x$$

$$\Rightarrow \boxed{y^2 = \frac{9}{2}x}$$

[Q.12] Vertex $(0,0)$

Symmetric about $\rightarrow$ y-axis

Passes through $\rightarrow (5,2)$

UPWARD

$$\boxed{x^2 = 4ay}$$

$A(5,2) \rightarrow$

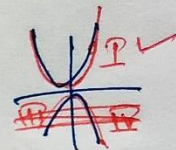
$$\Rightarrow (5)^2 = 4a(2)$$

$$\Rightarrow \boxed{\frac{25}{2} = 4a}$$

Parabola

$$\boxed{x^2 = \frac{25y}{2}}$$

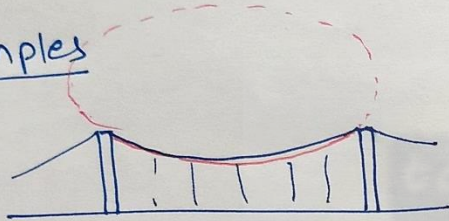
$$\boxed{2x^2 = 25y}$$



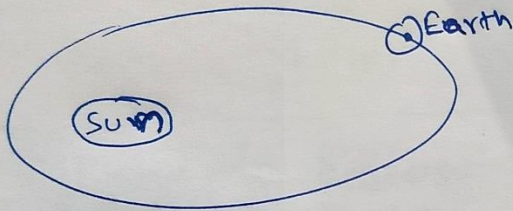
ELLIPSE (before Exercise-11.3)

Practical Examples

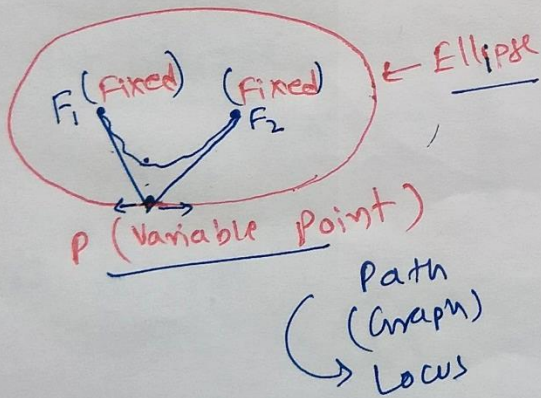
①



②

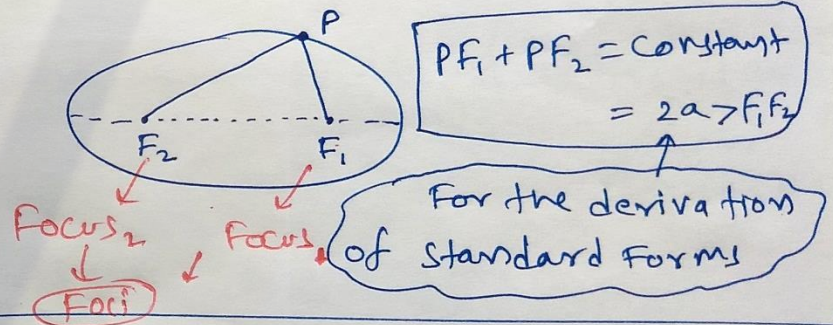


③

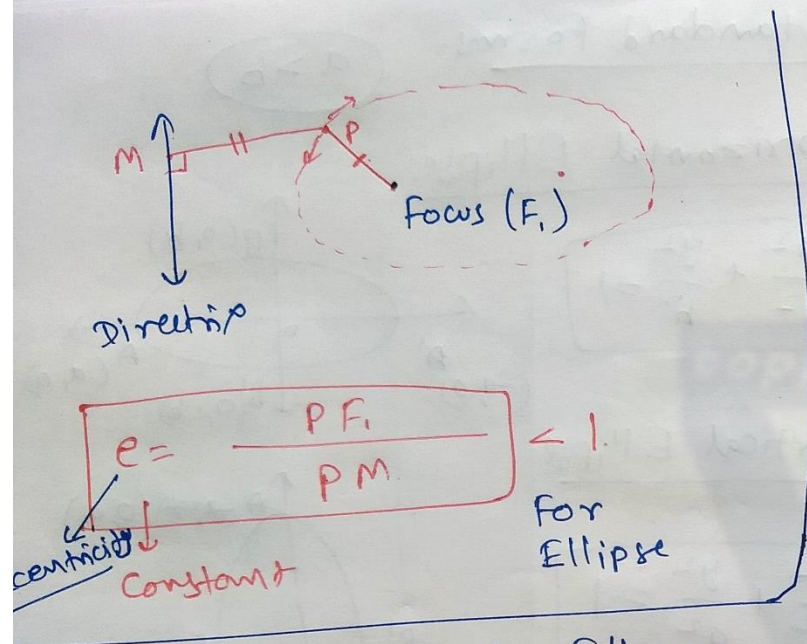


Definition - I.

Ellipse is the locus of a moving point (P) whose sum of distances ($PF_1 + PF_2$) from two fixed points (F_1 & F_2) is always constant.

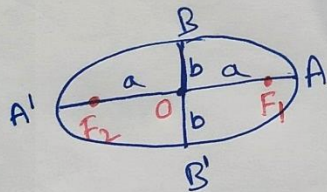


Definition - II Ellipse is the locus of a moving point (P) such that ratio of distance from P to a fixed point (F_1) & distance from P to a fixed line (Directrix) is constant (e) and always less than 1.

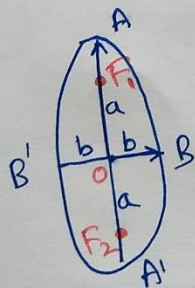


Terms Related to Ellipse

Horizontal Ellipse



Vertical Ellipse



$$\text{Major Axis} = AA' = 2a$$

$$a > b$$

$$\text{Minor Axis} = BB' = 2b$$

$$\text{Semi Major Axis} = OA = OA' = a$$

$$\text{Semi minor Axis} = OB = OB' = b$$

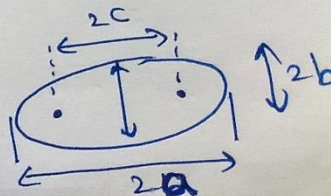
Centre (O) $\leftarrow$ Point of intersection of major & minor Axes.

Vertices $\rightarrow A \text{ \& } A'$ only

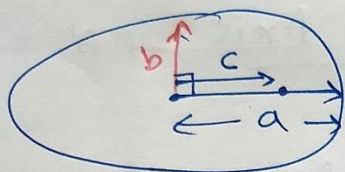
$\uparrow$
[end points of major Axis (AA')]

Distance b/w foci $= (F_1 F_2) = 2c$

$$OF_1 = OF_2 = c$$

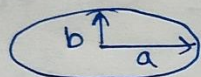


After Derivation



$$c = \sqrt{a^2 - b^2} \quad \star$$

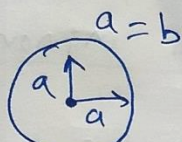
Special Case.



Ellipse

$$c = \sqrt{a^2 - b^2}$$

$a = b$



Circle.

$$c = 0$$

Eccentricity $= e = \frac{c}{a} < 1$

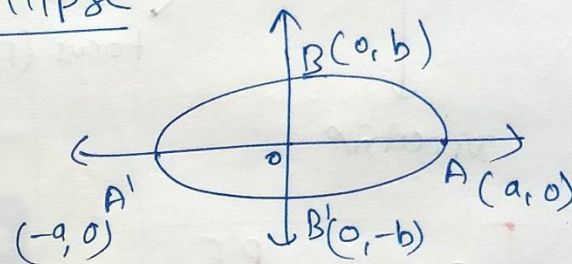
$a > b, c$ $e < 1$

Standard Forms.

$a > b$

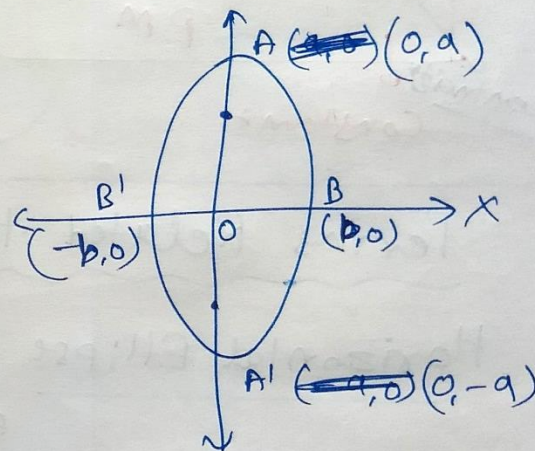
Horizontal Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



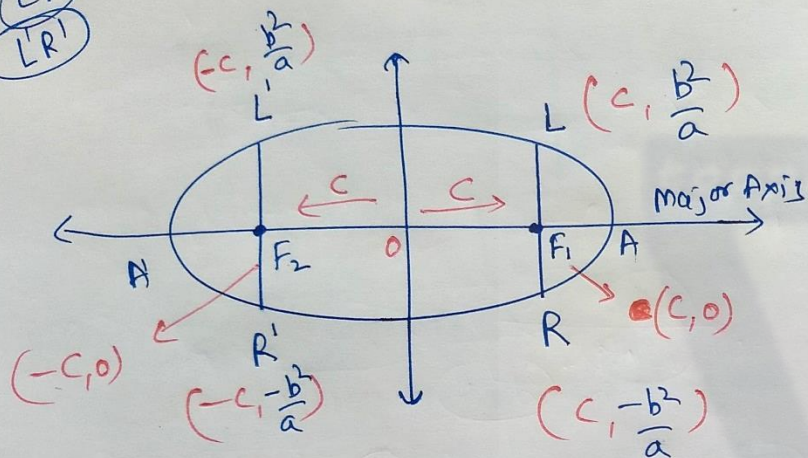
Vertical Ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



Latus Rectum. $\perp$ Major Axis
 (Passes through Focus)

LR
 LR'

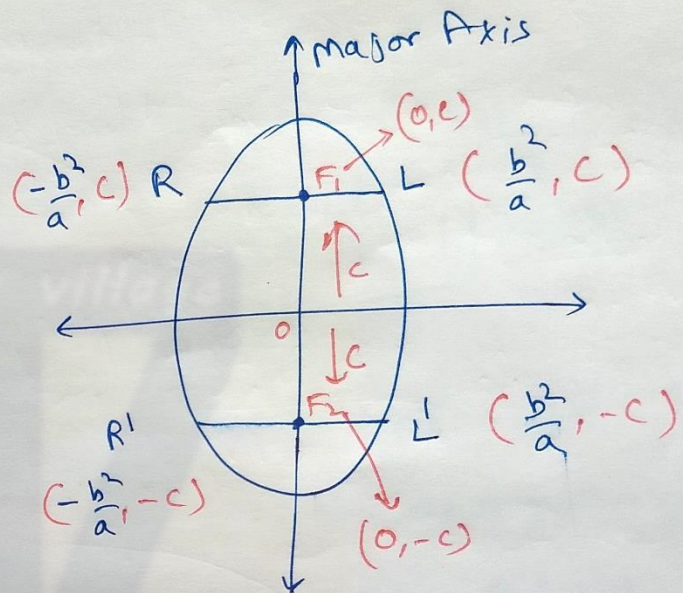


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a > b$

for L' $x = c$

Solve $y = \frac{b^2}{a}$



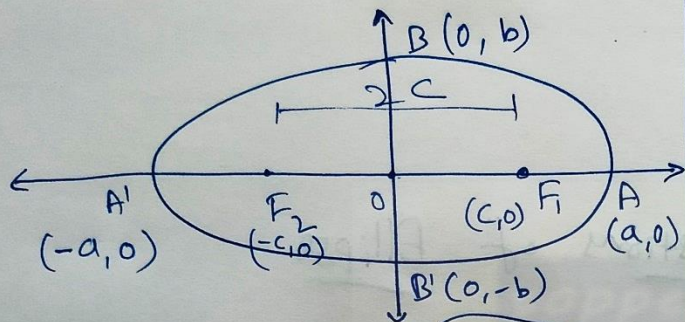
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad a > b$$

★ Length of Latus Rectum = $2 \frac{b^2}{a}$

$L(c, \frac{b^2}{a})$
 $R(c, -\frac{b^2}{a})$

Horizontal Ellipse

$$a > b$$



EQUATION $\Rightarrow$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Centre $\Rightarrow (0, 0)$

Major Axis $\Rightarrow$ x-axis $\rightarrow AA' \rightarrow 2a$

Minor Axis $\Rightarrow$ y-axis $\rightarrow BB' \rightarrow 2b$

Vertices $\Rightarrow$ ~~A, A'~~ A(a, 0), A'(-a, 0)

Foci $\Rightarrow F_1(c, 0), F_2(-c, 0)$

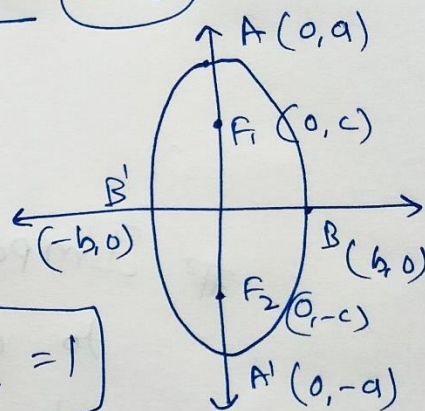
Length of Latus Rectum $= \frac{2b^2}{a}$

Eccentricity $= e = \frac{c}{a}$

$$c = \sqrt{a^2 - b^2}$$

Vertical Ellipse

$$a > b$$



EQUATION

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Centre $\Rightarrow (0, 0)$

Major Axis $\Rightarrow$ y-axis $\rightarrow AA' = 2a$

Minor Axis $\rightarrow$ x-axis $\rightarrow BB' = 2b$

Vertices $\rightarrow$ A(0, a), A'(0, -a)

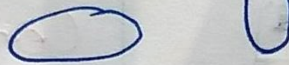
Foci $\rightarrow F_1(0, c), F_2(0, -c)$

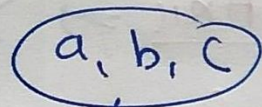
Length of Latus Rectum $= \frac{2b^2}{a}$

Eccentricity $= e = \frac{c}{a}$

$$c = \sqrt{a^2 - b^2}$$

Important things
to deal with Questions of Ellipse

Diagram 


↓

any two $(a, b), (b, c), (a, c)$

Q.1

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

larger

$$a > b$$

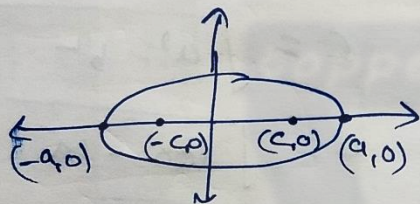
Horizontal Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a = 6, b = 4$$

$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{36 - 16} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$



$$\textcircled{1} \text{ Foci } (\pm c, 0) = (\pm 2\sqrt{5}, 0)$$

$$\textcircled{2} \text{ Vertices } (\pm a, 0) = (\pm 6, 0)$$

$$\textcircled{3} \text{ length of Major Axis} = 2a = 12$$

$$\textcircled{4} \text{ length of Minor Axis} = 2b = 8$$

$$\textcircled{5} \text{ Eccentricity } = e = \frac{c}{a} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$$

$$\textcircled{6} \text{ length of Latus Rectum} = \frac{2b^2}{a} = \frac{2 \times 16}{6} = \frac{16}{3}$$

Q.2

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

larger

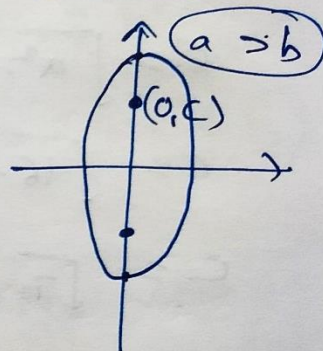
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$b = 2, a = 5$$

$$c = \sqrt{a^2 - b^2} = \sqrt{25 - 4}$$

$$c = \sqrt{21}$$

Vertical



$$\textcircled{1} \text{ Foci } \Rightarrow (0, \pm c) = (0, \pm \sqrt{21})$$

$$\textcircled{2} \text{ Vertices } \rightarrow (0, \pm a) = (0, \pm 5)$$

$$\textcircled{3} \text{ length of Major Axis} = 2a = 10$$

$$\textcircled{4} \text{ ————— minor Axis} = 2b = 4$$

$$\textcircled{5} e = \frac{c}{a} = \frac{\sqrt{21}}{5}$$

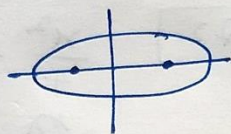
$$\textcircled{6} \text{ length of Latus Rectum} = \frac{2b^2}{a} = \frac{2 \times 4}{5} = \frac{8}{5}$$

Q.3 $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\downarrow$ $\nwarrow b^2$

(a^2) Larger $\rightarrow a = 4$
 $b = 3$

Horizontal



$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{16 - 9}$$

$$c = \sqrt{7}$$

① Foci $(\pm c, 0) = (\pm \sqrt{7}, 0)$

② Vertices $(\pm a, 0) = (\pm 4, 0)$

③ length of major axis $= 2a = 8$

④ ———— minor axis $= 2b = 6$

⑤ eccentricity $= e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

⑥ length of Latus Rectum $= \frac{2b^2}{a}$
 $= \frac{2 \times 9}{4} = \frac{9}{2}$

Q.4 $\frac{x^2}{25} + \frac{y^2}{100} = 1$

$\uparrow (b^2)$ $\rightarrow$ Larger $= a^2$

$$a = 10$$

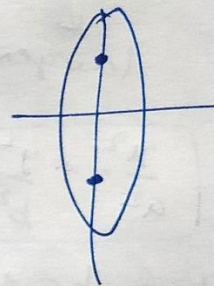
$$b = 5$$

$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{100 - 25}$$

$$c = \sqrt{3 \times 25} = 5\sqrt{3}$$

Vertical



① Foci $= (0, \pm c) = (0, \pm 5\sqrt{3})$

② vertices $= (0, \pm a) = (0, \pm 10)$

③ length of major axis $= 2a = 20$

④ ———— minor axis $= 2b = 10$

⑤ $e = \frac{c}{a} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$

⑥ length of Latus rectum $= 2 \frac{b^2}{a}$

$$= \frac{2 \times 25 \times 5}{10} = 5 \checkmark$$

Q.5

$$\frac{x^2}{49} + \frac{y^2}{36} = 1$$

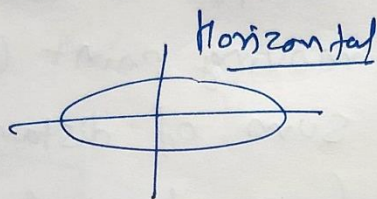
$\downarrow$
 Larger = a^2

$$a = 7$$

$$b = 6$$

$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{49 - 36} = \sqrt{13}$$



- ① Foci $\Rightarrow (\pm c, 0) \rightarrow (\pm \sqrt{13}, 0)$
- ② Vertices $\Rightarrow (\pm a, 0) \rightarrow (\pm 7, 0)$
- ③ length of major Axis = $2a = 14$
- ④ ——— minor Axis = $2b = 12$
- ⑤ Eccentricity = $e = \frac{c}{a} = \frac{\sqrt{13}}{7}$
- ⑥ Length of Latus Rectum = $\frac{2b^2}{a} = \frac{2(36)}{7} = \frac{72}{7}$

Q.6

$$\frac{x^2}{100} + \frac{y^2}{400} = 1$$

$\downarrow$
 Larger = a^2

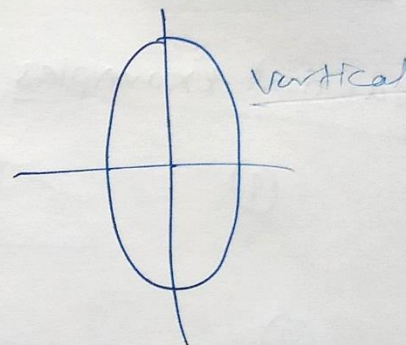
$$a = 20$$

$$b = 10$$

$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{300}$$

$$c = 10\sqrt{3}$$



- ① Foci $\Rightarrow (0, \pm c) = (0, \pm 10\sqrt{3})$
- ② Vertices $\Rightarrow (0, \pm a) = (0, \pm 20)$
- ③ length of major axis = $2a = 40$
- ④ ——— minor axis = $2b = 20$
- ⑤ $e = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$
- ⑥ length of latus rectum = $\frac{2b^2}{a} = \frac{2(100)}{20} = 10$

Q.7 $\frac{36x^2 + 4y^2}{144} = 1$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{36} = 1$



$b^2 = 9$
 $b = 3$

Larger $= a^2$

$a = 6$

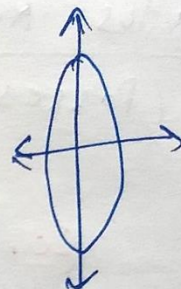
$c = \sqrt{a^2 - b^2} = \sqrt{36 - 9}$

$c = \sqrt{27} = 3\sqrt{3}$

- ① Foci $(0, \pm c) = (0, \pm 3\sqrt{3})$
- ② Vertices $(0, \pm a) = (0, \pm 6)$
- ③ length of major axis $= 2a = 12$
- ④ ——— minor axis $= 2b = 6$
- ⑤ Eccentricity $= e = \frac{c}{a} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$
- ⑥ length of Latus Rectum $= \frac{2b^2}{a} = \frac{2(9)}{6}$
 $= 3$

Q.8 ~~$\frac{16x^2 + y^2}{36} = 1$~~
 $\frac{16x^2 + y^2}{16} = 1$

$\Rightarrow \frac{x^2}{1} + \frac{y^2}{16} = 1$



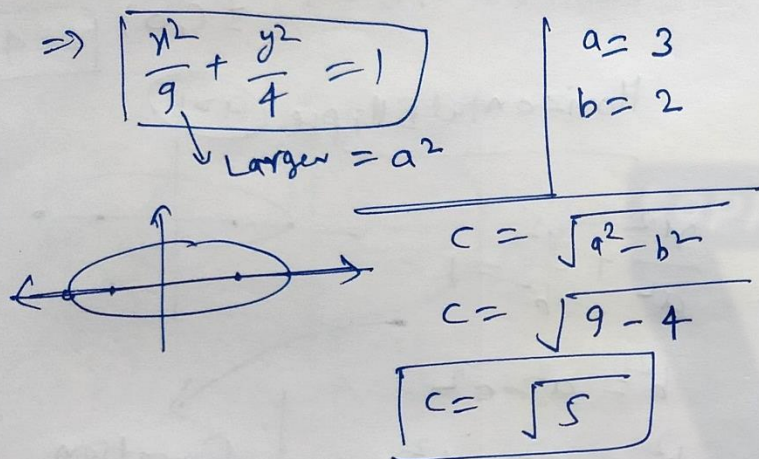
$a = 4$

$b^2 = 1$

$c = \sqrt{a^2 - b^2} = \sqrt{16 - 1} = \sqrt{15}$

- ① Foci $\rightarrow (0, \pm c) = (0, \pm \sqrt{15})$
- ② Vertices $\rightarrow (0, \pm a) = (0, \pm 4)$
- ③ Length of major axis $= 2a = 8$
- ④ ——— minor axis $= 2b = 2$
- ⑤ $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$
- ⑥ length of Latus Rectum $= \frac{2b^2}{a}$
 $= \frac{2(1)}{4} = \frac{1}{2}$

Q.9 $\frac{4x^2 + 9y^2}{36} = 1$



- ① Foci $(\pm c, 0) \rightarrow (\pm \sqrt{5}, 0)$
- ② Vertices $(\pm a, 0) = (\pm 3, 0)$
- ③ length of major axis $= 2a = 6$
- ④ ——— minor axis $= 2b = 4$
- ⑤ Eccentricity $= e = \frac{c}{a} = \frac{\sqrt{5}}{3}$
- ⑥ length of Latus Rectum $= \frac{2b^2}{a}$
 $= \frac{2(4)}{3} = \frac{8}{3}$

Toppers Village

Exercise 11.3 Ellipse

Q.10

Vertices $(\pm a, 0)$
 $(\pm 5, 0)$

foci $(\pm c, 0)$
 $(\pm 4, 0)$

y-coordinate
" 0

Ellipse $\rightarrow$ Horizontal,
 $a > b$

$\rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

$a = 5, b = 3$

Equation

$\boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$

$a = 5$
 $c = 4$

$c = \sqrt{a^2 - b^2}$
 $\Rightarrow c^2 = a^2 - b^2$
 $\Rightarrow b^2 = a^2 - c^2$
 $\Rightarrow b^2 = 25 - 16$
 $b^2 = 9$
 $b = 3$

[Q.11]

Vertices $(0, \pm 13)$ $(0, \pm a)$

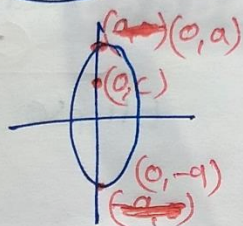
Foci $(0, \pm 5)$ $(0, \pm c)$

x-coordinates = 0

$a > b$

Ellipse $\rightarrow$ Vertical,

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



$a = 13, c = 5$

$$c = \sqrt{a^2 - b^2}$$

$$c^2 = a^2 - b^2$$

$$b^2 = a^2 - c^2$$

$$b^2 = 169 - 25 = 144$$

$b = 12$

$a = 13$
 $b = 12$

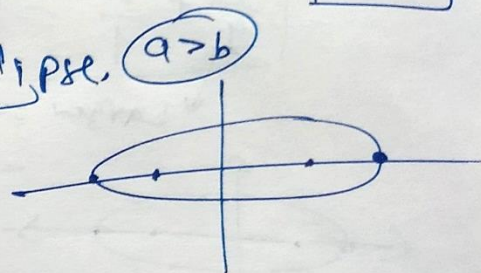
$$\frac{x^2}{144} + \frac{y^2}{169} = 1$$

[Q.12]

Vertices $(\pm 6, 0) \rightarrow (\pm a, 0)$ $a = 6$

foci $(\pm 4, 0) \rightarrow (\pm c, 0)$ $c = 4$

Horizontal Ellipse, $a > b$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2 = a^2 - c^2$$

$$b^2 = 6^2 - 4^2$$

$$b^2 = 36 - 16 = 20$$

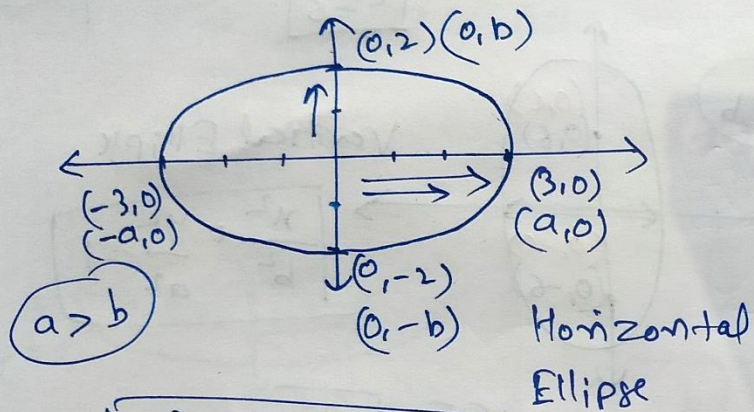
$$b = \sqrt{20} = 2\sqrt{5}$$

Equation

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

Q.13 Ends of major axis $(\pm 3, 0)$

Ends of minor axis $(0, \pm 2)$



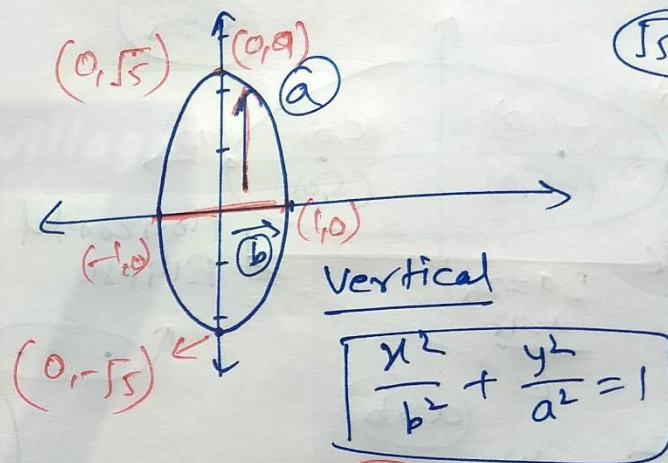
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\begin{cases} a=3 \\ b=2 \end{cases}$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \quad \checkmark$$

Q.14 Ends of major axis $(0, \pm \sqrt{5})$

Ends of minor axis $(\pm 1, 0)$



$\sqrt{5} \approx 2.236$

$a = \sqrt{5}$ $b = 1$

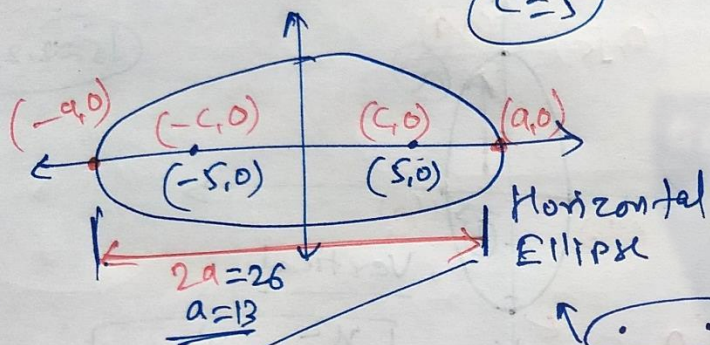
$$\Rightarrow \frac{x^2}{1^2} + \frac{y^2}{(\sqrt{5})^2} = 1$$

$$\Rightarrow \frac{x^2}{1} + \frac{y^2}{5} = 1$$

Q.15 length of major axis = 26 $\Rightarrow 2a$

$$\text{foci} = (\pm 5, 0) \equiv (\pm c, 0)$$

$$c = 5$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{169} + \frac{y^2}{144} = 1$$

$$a > b$$

$$a = 13$$

$$c = 5$$

$$b^2 = a^2 - c^2$$

$$b^2 = 169 - 25$$

$$b^2 = 144$$

$$c = \sqrt{a^2 - b^2}$$

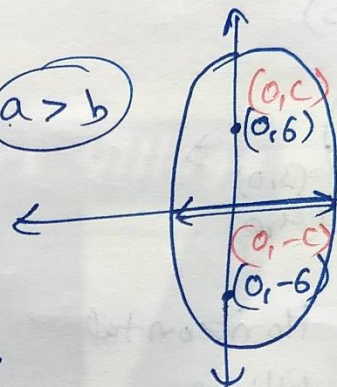
Q.16

length of minor axis = 16 = 2b

$$\text{foci} = (0, \pm 6) \equiv (0, \pm c) \quad \downarrow \quad b = 8$$

$$c = 6$$

$$a > b$$



Vertical Ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$c = \sqrt{a^2 - b^2}$$

$$\Rightarrow c^2 = a^2 - b^2$$

$$\Rightarrow a^2 = c^2 + b^2$$

$$\Rightarrow a^2 = 6^2 + 8^2$$

$$\Rightarrow a^2 = 36 + 64 = 100$$

$$a = 10$$

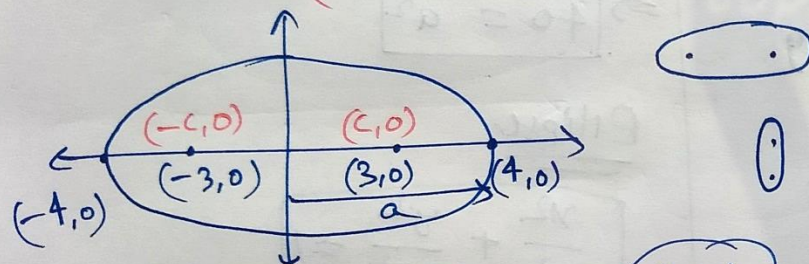
Equation of Ellipse

$$\frac{x^2}{64} + \frac{y^2}{100} = 1$$

Toppers Village.

Exercise 11.3 Ellipse

[Q.17] Foci $(\pm 3, 0)$, $a = 4$
 $(\pm c, 0)$



Horizontal Ellipse

$$a > b$$

$$c = 3 \quad (a = 4)$$

$$c = \sqrt{a^2 - b^2}$$

$$\Rightarrow b^2 = a^2 - c^2$$

$$\Rightarrow b^2 = 16 - 9$$

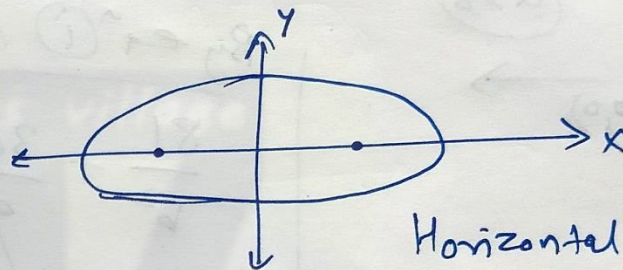
$$b^2 = 7$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

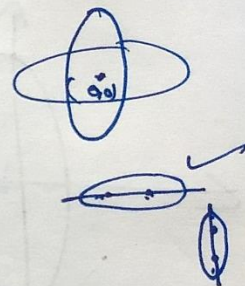
$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

[Q.18] $b = 3$, $c = 4$, Centre $(0, 0)$

Foci on x-axis



Horizontal



$$a > b$$

$$b = 3$$

$$c = 4$$

$$c = \sqrt{a^2 - b^2}$$

$$\Rightarrow c^2 = a^2 - b^2$$

$$\Rightarrow c^2 + b^2 = a^2$$

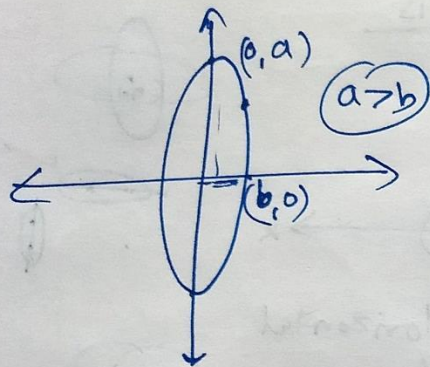
$$\Rightarrow 16 + 9 = a^2$$

$$\Rightarrow a^2 = 25$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Q.19 Centre $(0,0)$, Major Axis = y-axis
 Passes through $(3,2)$ & $(1,6)$



Vertical Ellipse

$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

① $(3,2)$ ↑

$$\Rightarrow \boxed{\frac{9}{b^2} + \frac{4}{a^2} = 1} \times \textcircled{9}$$

② $(1,6)$

$$\boxed{\frac{1}{b^2} + \frac{36}{a^2} = 1} \textcircled{2}$$

By eqⁿ ① & ②:→

$$\frac{81}{b^2} + \frac{36}{a^2} = 9$$

$$\frac{1}{b^2} + \frac{36}{a^2} = 1$$

$$\frac{1080}{b^2} = 8$$

$$\Rightarrow \boxed{10 = b^2}$$

By eqⁿ ②:

$$\frac{1}{10} + \frac{36}{a^2} = 1$$

$$\Rightarrow \frac{36}{a^2} = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\Rightarrow \frac{432}{a^2} = \frac{9}{10}$$

$$\Rightarrow \boxed{40 = a^2}$$

Ellipse

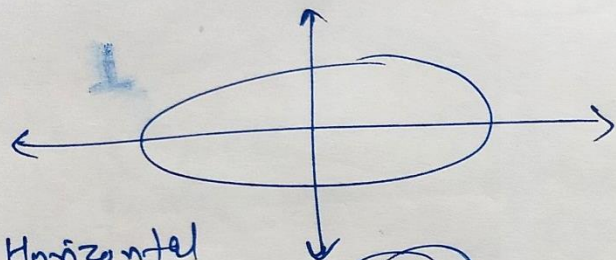
$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

$$\Rightarrow \boxed{\frac{x^2}{10} + \frac{y^2}{40} = 1} \checkmark$$

Q.20

Major Axis $\rightarrow$ x-axis

Passes through (4,3) & (6,2)



Horizontal

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Large Small

$$a > b$$

① (4,3) $\rightarrow$ $\left(\frac{16}{a^2} + \frac{9}{b^2} = 1\right)$ — ① $\times 4$

② (6,2) $\rightarrow$ $\left(\frac{36}{a^2} + \frac{4}{b^2} = 1\right)$ — ② $\times 9$

By eqⁿ ① & ②

$$\frac{64}{a^2} + \frac{36}{b^2} = 4$$

$$\frac{324}{a^2} + \frac{36}{b^2} = 9$$

$$\Rightarrow \frac{64 - 324}{a^2} = 4 - 9$$

$$\Rightarrow \frac{-260}{a^2} = -5$$

$$\Rightarrow a^2 = 52$$

By eqⁿ ②

$$\frac{36}{52} + \frac{4}{b^2} = 1$$

$$\Rightarrow \frac{9}{13} + \frac{4}{b^2} = 1$$

$$\frac{4}{b^2} = 1 - \frac{9}{13}$$

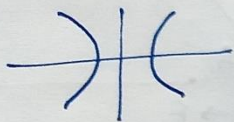
$$\Rightarrow \frac{4}{b^2} = \frac{13 - 9}{13} = \frac{4}{13}$$

$$\Rightarrow b^2 = 13$$

$$\frac{x^2}{52} + \frac{y^2}{13} = 1$$

Hyperbola

before Exercise (11.4)



Real life Examples.

① Nuclear Reactor



② Chair



③



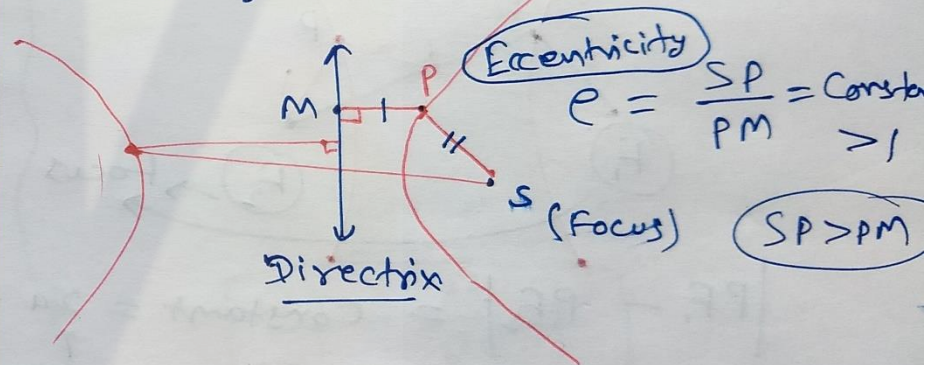
④



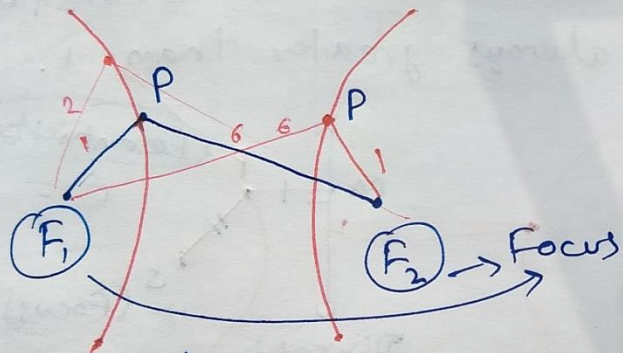
⑤



Definition - I Hyperbola is the locus of a moving point 'P' such that the ratio of distance b/w 'P' & a fixed point 'S' (Focus) & a fixed line (Directrix) is constant and always greater than 1.



Definition - ② Hyperbola is the locus of a moving point (P) from where difference of distances to the two fixed points (foci- F_1 & F_2) is always constant.

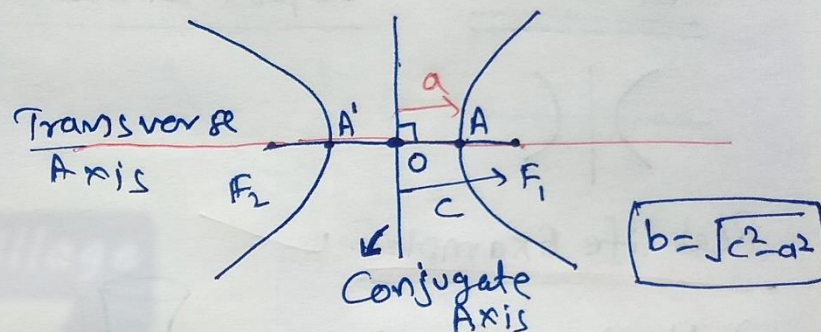


$$|PF_1 - PF_2| = \text{Constant} = 2a$$

↑
(for Derivation)

$$|1 - 6| = |6 - 1|$$

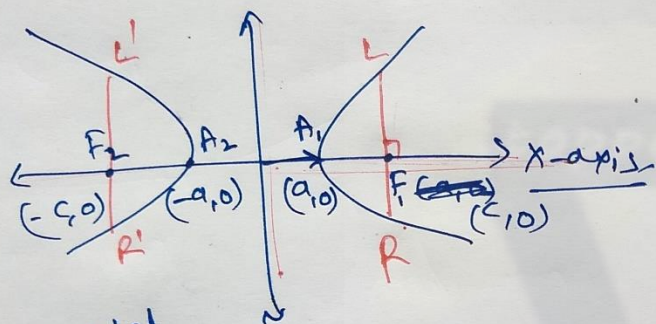
Terms Related to HYPERBOLA



- ① Foci (F_1 & F_2) $(c, 0), (-c, 0)$
- ② Centre. $F_1, F_2 \xleftarrow{\text{mid point } O}$
- ③ Transverse Axis -
- ④ Conjugate Axis -
- ⑤ Vertices A & A' $(a, 0)$ & $(-a, 0)$
- ⑥ eccentricity $= e = \frac{c}{a}$

Equations of Hyperbola : (standard)

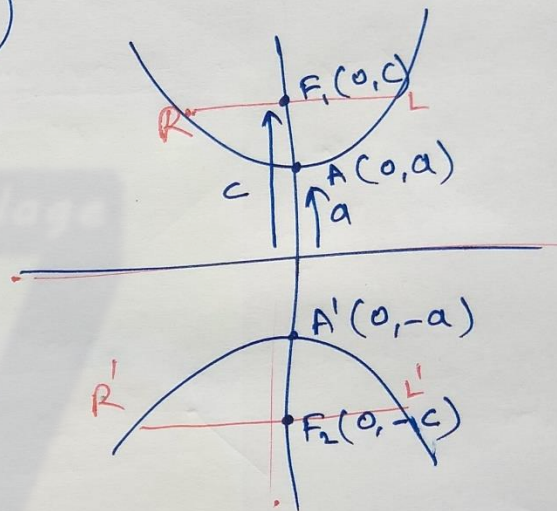
(I)



Horizontal
Hyp.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(II)



Vertical Hyp.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

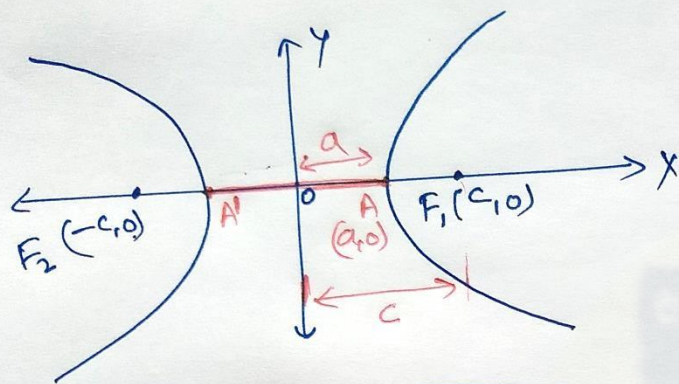
Note: If $a = b$

$$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$$

$$\Rightarrow \boxed{x^2 - y^2 = a^2}$$

Equilateral
Hyperbola (Rectangular
Hyperbola)

Horizontal Hyperbola



EQUATION

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Centre (0,0)

Transverse Axis x-axis $\rightarrow AA' = 2a$

Conjugate Axis y-axis $\rightarrow 2b$

Vertices A & A' ($\pm a, 0$)

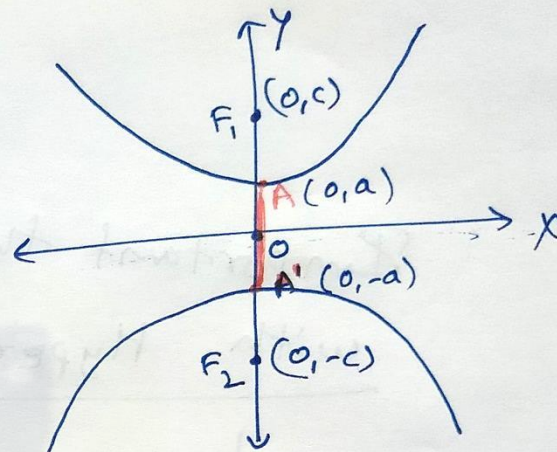
Foci F_1, F_2 ($\pm c, 0$)

length of latus Rectum = $\frac{2b^2}{a}$

$$e = \frac{c}{a}$$

$$c = \sqrt{a^2 + b^2} \rightarrow b = \sqrt{c^2 - a^2}$$

Vertical Hyperbola



EQUATION

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Centre (0,0)

Transverse Axis $\rightarrow$ y-axis $\rightarrow 2a = AA'$

Conjugate Axis $\rightarrow$ x-axis $\rightarrow 2b$

Vertices A & A' ($0, \pm a$)

Foci F_1, F_2 ($0, \pm c$)

length of latus Rectum = $\frac{2b^2}{a}$

$$e = \frac{c}{a}$$

$$c = \sqrt{a^2 + b^2} \rightarrow b = \sqrt{c^2 - a^2}$$

Important things to deal with Hyperbola Questions

- Diagram
- any two out of a, b, c

Exercise 11.4 (Hyperbola)

Q.1 $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Horizontal Hyp. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

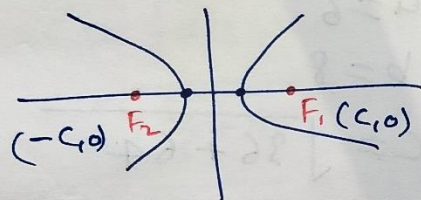
$a=4, b=3$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{16 + 9}$

$c = \sqrt{25}$

$c = 5$



coordinates of foci $(F_1 \text{ \& } F_2) = (\pm c, 0) \rightarrow (\pm 5, 0)$

Vertices $(\pm a, 0) = (\pm 4, 0)$

Eccentricity $= e = \frac{c}{a} = \frac{5}{4}$

length of Latus Rectum $= \frac{2b^2}{a} = \frac{2(9)}{4}$
 $= \frac{9}{2}$ units

Q.2 $\frac{y^2}{9} - \frac{x^2}{27} = 1$

$\oplus \rightarrow$ Vertical Hyp. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$a=3$

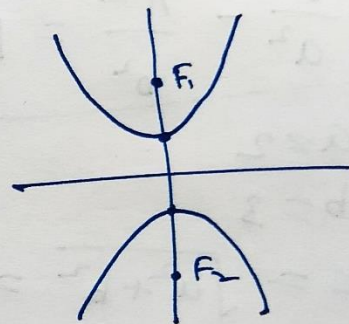
$b=3\sqrt{3}$

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{9 + 27}$

$c = \sqrt{36}$

$c = 6$



Foci $(0, \pm c) \rightarrow (0, \pm 6)$

Vertices $(0, \pm a) \rightarrow (0, \pm 3)$

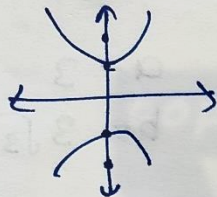
Eccentricity $= e = \frac{c}{a} = \frac{6}{3} = 2$

length of Latus Rectum $= \frac{2b^2}{a} = \frac{2(27)}{3}$
 $= 18$ ✓

Q.3 $\frac{9y^2 - 4x^2}{36} = 1$

$\Rightarrow \frac{y^2}{4} - \frac{x^2}{9} = 1$ ✓ Vertical Hyp.

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ ✓



$a = 2$

$b = 3$

$c = \sqrt{a^2 + b^2} = \sqrt{4 + 9} = \sqrt{13}$

Foci $(0, \pm c) \equiv (0, \pm \sqrt{13})$

Vertices $(0, \pm a) \equiv (0, \pm 2)$

Eccentricity $= e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

length of Latus Rectum $= \frac{2b^2}{a} = \frac{2(9)}{2} = 9$

Q.4 $\frac{16x^2 - 9y^2}{576} = 1$

$\Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1$

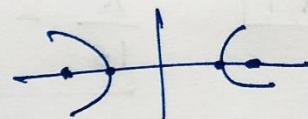
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$a = 6$

$b = 8$

$c = \sqrt{36 + 64} = \sqrt{100} = 10$

Horizontal



Foci $\rightarrow (\pm c, 0) \equiv (\pm 10, 0)$

Vertices $\rightarrow (\pm a, 0) \equiv (\pm 6, 0)$

Eccentricity $= e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$ ✓

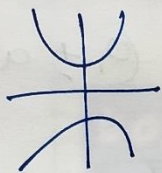
length of Latus Rectum $= \frac{2b^2}{a} = \frac{2(64)}{\frac{5}{3}} = \frac{64}{3}$ ✓

Q.5 $5y^2 - 9x^2 = 36$

$$\Rightarrow \frac{5y^2}{36} - \frac{x^2}{4} = 1$$

Vertical

$$\Rightarrow \frac{y^2}{\left(\frac{36}{5}\right)} - \frac{x^2}{(4)} = 1$$



$$\frac{y^2}{(a^2)} - \frac{x^2}{(b^2)} = 1$$

$$a = \frac{6}{\sqrt{5}}, b = 2$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{\frac{36}{5} + 4}$$

$$c = \sqrt{\frac{36+20}{5}} = \sqrt{\frac{56}{5}}$$

Foci $(\pm c, 0) \rightarrow (\pm \sqrt{\frac{56}{5}}, 0)$

Vertices $(\pm a, 0) \rightarrow (\pm \frac{6}{\sqrt{5}}, 0)$

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{\frac{56}{5}}}{\frac{6}{\sqrt{5}}} = \frac{\sqrt{56}}{6}$

Length of Latus Rectum $= \frac{2b^2}{a}$

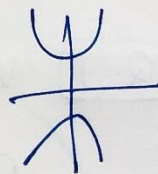
$$= \frac{2(4)}{\frac{6}{\sqrt{5}}} = \frac{4\sqrt{5}}{3}$$

Q.6 $49y^2 - 16x^2 = 784$

$$\Rightarrow \frac{y^2}{16} - \frac{x^2}{49} = 1$$

Vertical Hyp.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



$$a = 4$$

$$b = 7$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{16 + 49} = \sqrt{65}$$

Foci $(0, \pm c) \equiv (0, \pm \sqrt{65})$

Vertices $(0, \pm a) \equiv (0, \pm 4)$

Eccentricity $e = \frac{c}{a} = \frac{\sqrt{65}}{4}$

Length of Latus Rectum $= \frac{2b^2}{a} = \frac{2(49)}{4} = \frac{49}{2}$

Exercise 11.4 Hyperbola

Q.7 Vertices $(\pm 2, 0)$

Foci $(\pm 3, 0)$

$y=0$

Horizontal Hyp.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Equation

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

$$b = \sqrt{c^2 - a^2}$$

$$b = \sqrt{3^2 - 2^2}$$

$$b = \sqrt{9 - 4} = \sqrt{5}$$

$(\pm a, 0)$ $a=2$

$(\pm c, 0)$ $c=3$

Q.8 Vertices $(0, \pm 5)$

Foci $(0, \pm 8)$

$(0, \pm a)$ $a=5$

$(0, \pm c)$ $c=8$

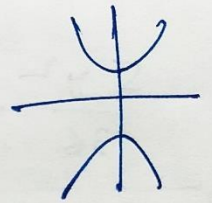
$$b = \sqrt{c^2 - a^2}$$

$$b = \sqrt{64 - 25}$$

$$b = \sqrt{39}$$

$$b^2 = 39$$

Vertical Hyp.



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Equation

$$\frac{y^2}{25} - \frac{x^2}{39} = 1$$

Q.9

Vertices $(0, \pm 3) \equiv (0, \pm a)$

Foci $(0, \pm 5) \equiv (0, \pm c)$

Vertical Hyp.

$a=3$

$c=5$

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$b = \sqrt{c^2 - a^2}$

$b = \sqrt{25 - 9}$

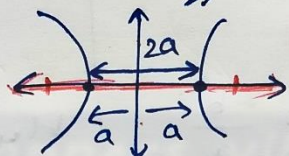
$b = \sqrt{16} = 4$

Eqⁿ: $\frac{y^2}{9} - \frac{x^2}{16} = 1$

Q.10 Foci $(\pm 5, 0) \equiv (\pm c, 0)$

length (transverse) = 8 = 2a
Axis

Hyp.



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$c=5$

$a=4$

$b = \sqrt{c^2 - a^2} = \sqrt{25 - 16}$

$b = \sqrt{9} = 3$

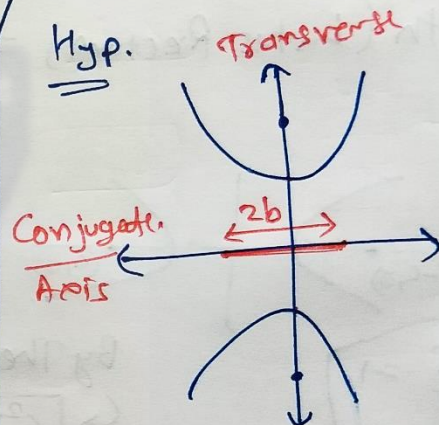
Eqⁿ: $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Q.11

Foci $(0, \pm 13) \equiv (0, \pm c)$

length (conjugate) = 24 = 2b
Axis

Hyp.



$c = \sqrt{a^2 + b^2}$

$c=13$

$b=12$

Eqⁿ: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$c^2 = a^2 + b^2$

$\Rightarrow c^2 - b^2 = a^2$

$\Rightarrow a^2 = 169 - 144$

$\Rightarrow a^2 = 25$

Eqⁿ: $\frac{y^2}{25} - \frac{x^2}{144} = 1$

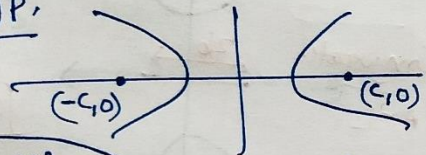
Toppers Village.

Exercise 11.4 Hyperbola

Q.12 Foci $(\pm 3\sqrt{5}, 0) \equiv (\pm c, 0)$

length (Latus Rectum) $= 8 = \frac{2b^2}{a}$
 $4a = b^2$

Hyp.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$a = 5$ ✓

$b^2 = 4a$

$b^2 = 20$ ✓

By Theory

$$c^2 = a^2 + b^2$$

$$\Rightarrow (3\sqrt{5})^2 = a^2 + 4a$$

$$\Rightarrow a^2 + 4a - 45 = 0$$

$$\Rightarrow a^2 + 9a - 5a - 45 = 0$$

$$\Rightarrow a(a+9) - 5(a+9) = 0$$

$$\Rightarrow (a-5)(a+9) = 0$$

$a = 5$

$a = -9$

Egⁿ.

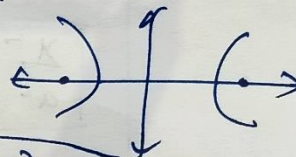
$$\frac{x^2}{25} - \frac{y^2}{20} = 1$$

Q.13

Foci $(\pm 4, 0) \equiv (\pm c, 0)$ $c = 4$

length (Latus Rectum) $= 12 = \frac{2b^2}{a}$

Hyp.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Egⁿ.

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

$$6a = b^2$$

$$c^2 = a^2 + b^2$$

$$\Rightarrow 16 = a^2 + 6a$$

$$\Rightarrow a^2 + 6a - 16 = 0$$

$$\Rightarrow a^2 + 8a - 2a - 16 = 0$$

$$\Rightarrow (a+8)(a-2) = 0$$

$a = -8$ $a = 2$

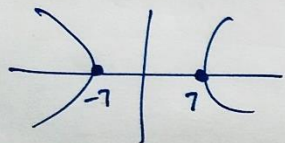
$a = 2$ ✓

$b^2 = 12$ ✓

Q.14 Vertices $(\pm 7, 0)$ $(\pm 9, 0)$

$$e = \frac{4}{3} = \frac{c}{a}$$

Hyp.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Equation

$$\frac{x^2}{49} - \frac{y^2}{343} = 1$$

$a = 7$

$$c = \frac{4a}{3}$$

$$c = \frac{4 \times 7}{3} = \frac{28}{3}$$

$$b = \sqrt{c^2 - a^2}$$

$$b^2 = \left(\frac{28}{3}\right)^2 - 49$$

$$b^2 = \frac{784}{9} - 49$$

$$b^2 = \frac{784 - 441}{9}$$

$$b^2 = \frac{343}{9}$$

Q.15 Foci $(0, \pm \sqrt{10}) \rightarrow (0, \pm c)$

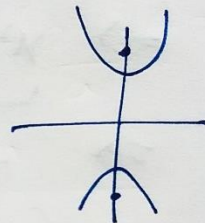
$$c = \sqrt{10}$$

Passing through $(2, 3)$

Vertical Hyp.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Put $(2, 3) \rightarrow$



$$\Rightarrow \frac{9}{a^2} - \frac{4}{b^2} = 1$$

$$c^2 = a^2 + b^2 \Rightarrow 10 = a^2 + b^2$$

Substitute

$$\Rightarrow a^2 = 10 - b^2 \quad \text{--- (1)}$$

$$\Rightarrow \frac{9}{10 - b^2} - \frac{4}{b^2} = 1$$

$$b^2 = x$$

$$\Rightarrow \frac{9}{10 - x} - \frac{4}{x} = 1$$

$$\Rightarrow \frac{9x - (40 - 4x)}{x(10 - x)} = 1$$

$$\Rightarrow 13x - 40 = 10x - x^2$$

$$\Rightarrow x^2 + 3x - 40 = 0$$

$$\Rightarrow x^2 + 3x - 40 = 0$$

$$(x = b^2)$$

$$\Rightarrow x^2 + 8x - 5x - 40 = 0$$

$$\Rightarrow x(x+8) - 5(x+8) = 0$$

$$(x-5)(x+8) = 0$$

$$b^2 = x = 5$$

$$b^2 = x = -8$$

$$b^2 = 5$$

By eqn (1) $a^2 = 10 - (5)$

$$a^2 = 5$$

Eqⁿ of Hyper.

$$\frac{y^2}{(5)} - \frac{x^2}{(5)} = 1$$

✓✓

Toppers Village

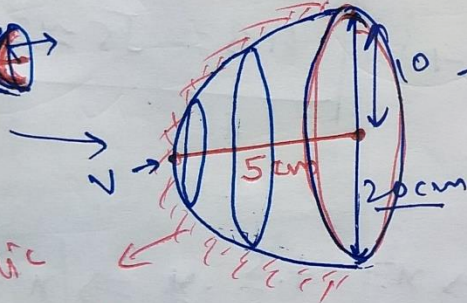
Miscellaneous Exercise - Chapter 11

Conic Sections

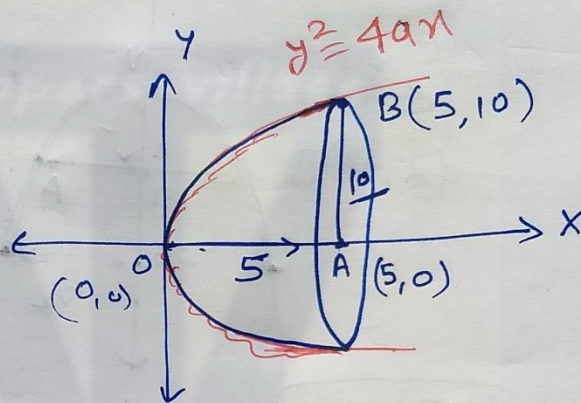
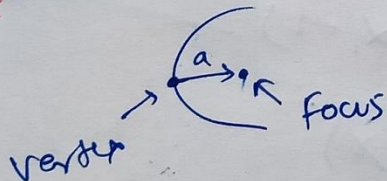
Q.1 Parabolic Reflector (Torch)

Diameter = 20 cm

Deep 5 cm



Parabolic
Reflector
Surface



$\therefore B(5,10)$ lies on Parabola $y^2 = 4ax$

$$\Rightarrow 100 = 4 \cdot a \cdot 5$$

$$\Rightarrow \frac{100}{20} = a$$

$$\Rightarrow \boxed{a = 5} \checkmark$$

$$\boxed{y^2 = 20x}$$

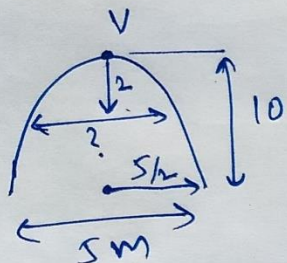
vertex (0,0)

Focus (5,0) $\rightarrow$ on A $\checkmark$

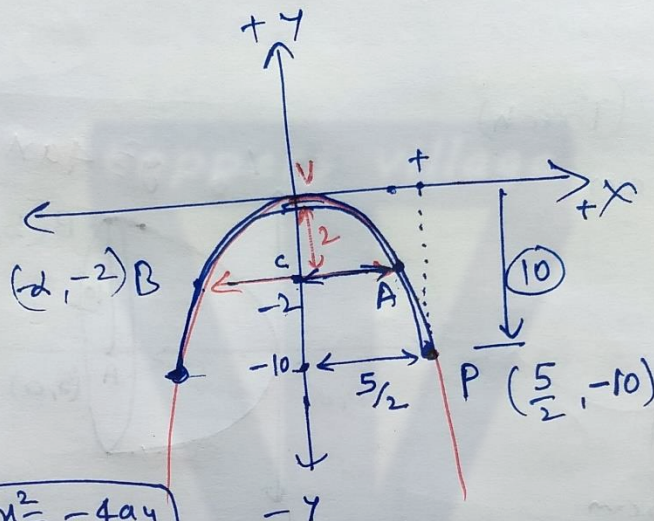
Q.2 Parabolic Arch

Axis Vertical

10 m high, 5 m wide at Base



How wide is it 2 m from vertex?



$$A(x, -2) \equiv A\left(\frac{\sqrt{5}}{2}, -2\right)$$

$$CA = \frac{\sqrt{5}}{2}$$

$$AB = 2 \times CA$$

$$AB = 2 \times \frac{\sqrt{5}}{2}$$

$$AB = \sqrt{5} \approx 2.236 \text{ m}$$

$\therefore P\left(\frac{5}{2}, -10\right)$ lies on

$$x^2 = -4ay$$

$$\Rightarrow \frac{5^2}{4} = +4a\left(\frac{-10}{2}\right)$$

$$\Rightarrow \frac{5}{32} = a$$

For Point A $\rightarrow x^2 = -4 \cdot \frac{5}{32} \cdot y \Rightarrow x^2 = -\frac{5}{8}y$

Coordinate $(x, -2)$

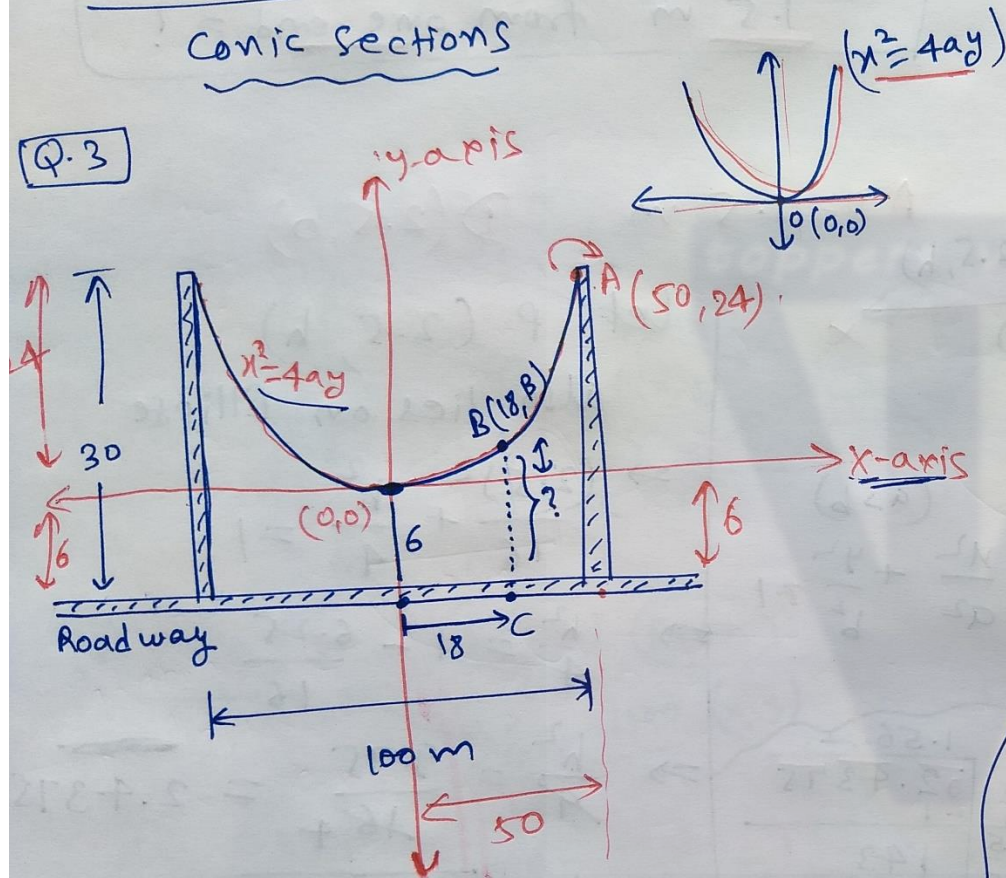
$$x^2 = +\frac{5}{8} \left(\frac{-x}{4}\right)$$

$$x^2 = \frac{5}{4} \Rightarrow x = \pm \frac{\sqrt{5}}{2}$$

Miscellaneous Exercise Chapter (11)

Conic Sections

Q.3



$A(50, 24)$ lies on $x^2 = 4ay$

$$\Rightarrow (50)^2 = 4a(24)$$

$$\Rightarrow \frac{625}{24} = a$$

$$\Rightarrow a = \frac{625}{24}$$

Parabola $x^2 = 4ay = 4 \times \frac{625}{24} \cdot y$

$$\Rightarrow x^2 = \frac{625}{6} y$$

$B(?) \rightarrow$ let B be $(18, B)$
Satisfy $(x^2 = \frac{625}{6} y)$

$$\Rightarrow 324 = \frac{625}{6} \cdot B$$

$$\Rightarrow B = \frac{324 \times 6}{625} \approx 3.1104 \text{ m}$$

$$B(18, 3.1104)$$

$$BC = y \text{ coordinate of } B + '6'$$

$$= 3.1104 + 6$$

$$BC = 9.1104 \text{ m}$$

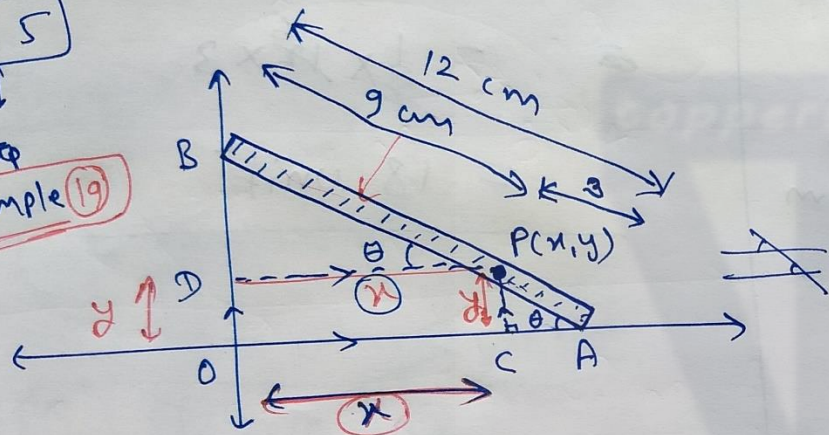
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Miscellaneous Exercise • Chapter (11)

Conic Sections

Q. 5

Example (19)



Let $P(x, y)$

Locus Path
 $\downarrow$
Equation (x, y)

In $\triangle ACP$ $\sin \theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{CP}{AP} = \frac{y}{3}$

$\Rightarrow \cancel{y = 3 \sin \theta}$

In $\triangle PDB$

$$\cos \theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{PD}{PB} = \frac{x}{9}$$

arbitrary constant
 $\updownarrow$
assume

$$\sin \theta = \frac{y}{3}, \quad \cos \theta = \frac{x}{9}$$

Identity $\sin^2 \theta + \cos^2 \theta = 1$

$$\Rightarrow \left(\frac{y}{3}\right)^2 + \left(\frac{x}{9}\right)^2 = 1$$

$$\Rightarrow \left[\frac{x^2}{81} + \frac{y^2}{9} = 1 \right] \checkmark$$

$P(x, y)$ Ellipse

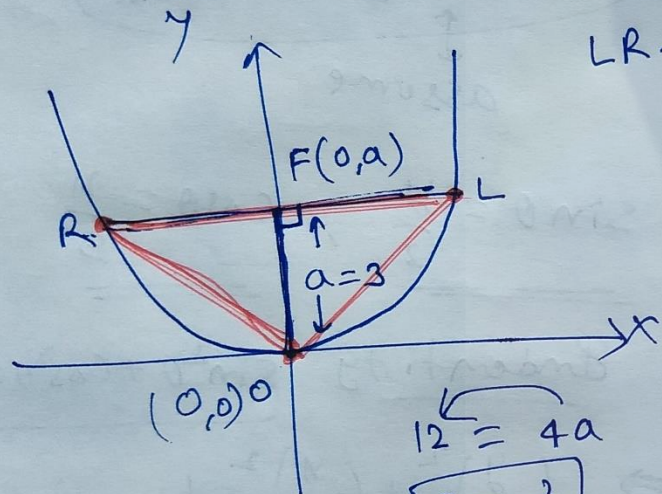
Q.6

$$x^2 = 12y$$



$$x^2 = 4ay$$

Triangle area $\rightarrow$ 3 point $\left\{ \begin{array}{l} \text{Latus Rectum} \\ \text{Vertex} \end{array} \right.$
 Δ



Focus $(0, a) \equiv (0, 3)$

LR = length of latus Rectum $= 4a = 12$

$$\boxed{LR = 12}$$

area of $\Delta OLR = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times LR \times OF$$

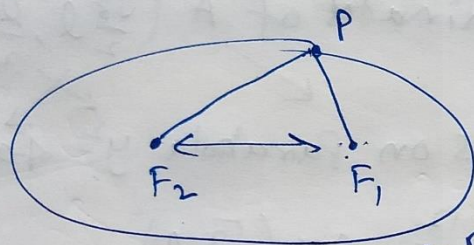
$$= \frac{1}{2} \times 12 \times 3$$

$$= 18 \text{ unit}^2$$

Miscellaneous Exercise. Chapter (11)

Conic Sections

Q.7

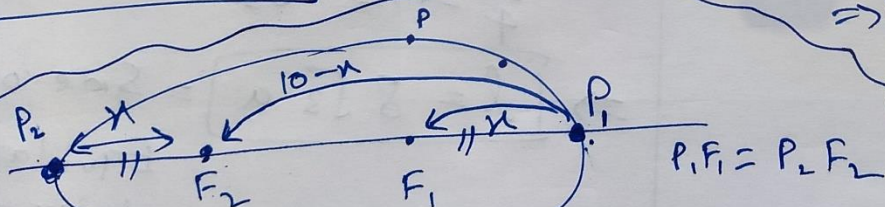


Ellipse

$$PF_1 + PF_2 = 10 = 2a$$

$$F_1F_2 = 8$$

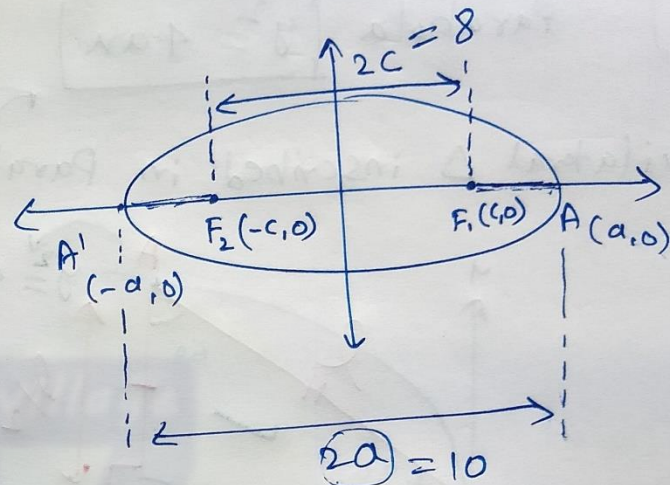
$$PF_1 + PF_2 = 2a$$



$$P_2P_1 = x + 10 - x = 10$$

$$PF_1 = x$$

$$PF_2 = 10 - x$$



$$a = 5$$

$$c = 4$$

$$\begin{aligned} \therefore c &= \sqrt{a^2 - b^2} \\ \Rightarrow c^2 &= a^2 - b^2 \\ \Rightarrow b^2 &= a^2 - c^2 \\ \Rightarrow b^2 &= 25 - 16 \\ \Rightarrow b^2 &= 9 \end{aligned}$$

Equation:

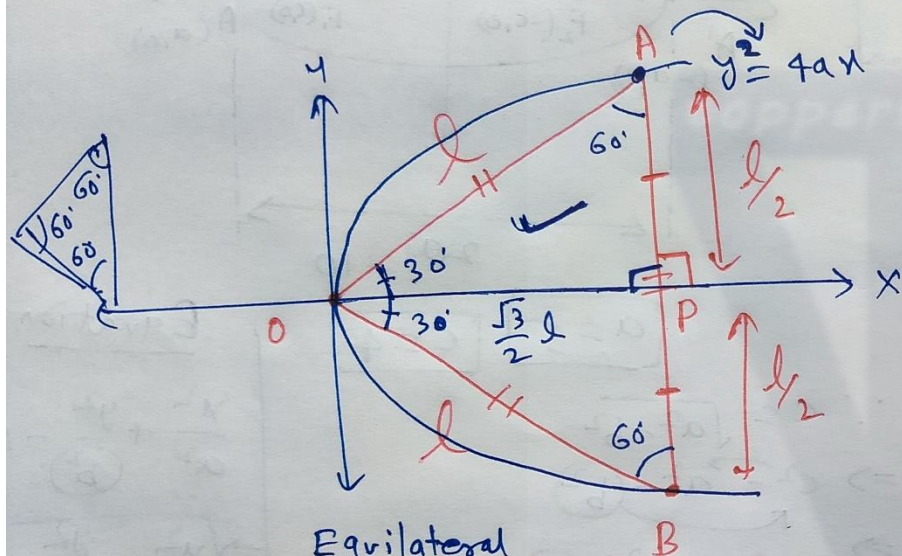
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Q.8

Parabola $y^2 = 4ax$

Equilateral Δ inscribed in Parabola



Equilateral
Side of $\Delta OAB = l$

In ΔOPA : $\cos 30^\circ = \frac{\text{Base}}{\text{Hyp.}}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OP}{OA}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OP}{l}$$

$$\Rightarrow OP = \frac{\sqrt{3}}{2} l$$

Coordinates of $A \left(\frac{\sqrt{3}}{2} l, \frac{l}{2} \right)$

lies on Parabola $y^2 = 4ax$

$$\Rightarrow \left(\frac{l}{2} \right)^2 = 4a \left(\frac{\sqrt{3}}{2} l \right)$$

$$\Rightarrow \frac{l^2}{4} = 4a \cdot \frac{\sqrt{3}}{2} l$$

$$\Rightarrow \frac{l}{4} = 2a\sqrt{3}$$

$$\Rightarrow \boxed{l = 8\sqrt{3}a} = \text{Side of the Equilateral triangle}$$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>