

TOPPERS VILLAGE



Class 11 Mathematics

Chapter 13: Limits and Derivatives

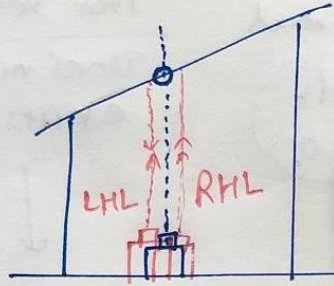
Class Notes (pdf)

YouTube पर इस Chapter के FREE Videos देखिए ... (Playlist की Link नीचे है) ↓↓↓

<https://www.youtube.com/watch?v=Ayb5qJxebA&list=PLPFrn0ppwwkQpZpaAeaDQ65Pcm-krcXQ8>

Limit:

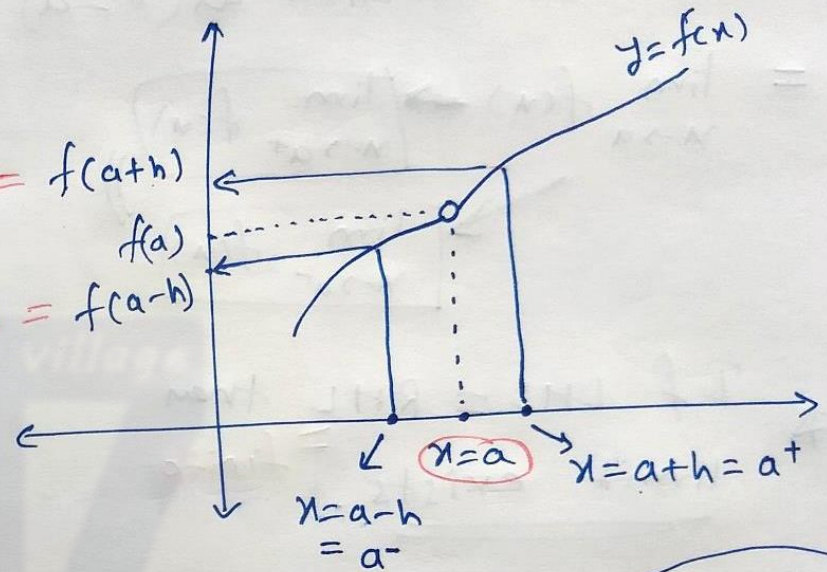
machine.
 $d = S \times t$



Definition! $\rightarrow$ limit of a function $y = f(x)$ at $x = a$ is the value of function in the neighborhood of $x = a$.

$$RHL = f(a+h)$$

$$LHL = f(a-h)$$



$$h \approx 0 \quad (h = 0.0000001)$$

$$h \rightarrow 0$$

$$RHL = \text{Right Hand limit} = \lim_{x \rightarrow a^+} f(x)$$

$$= \lim_{x \rightarrow a+h} f(x) = f(a^+) = f(a+h)$$

$$LHL = \text{Left Hand limit} = \lim_{x \rightarrow a^-} f(x)$$

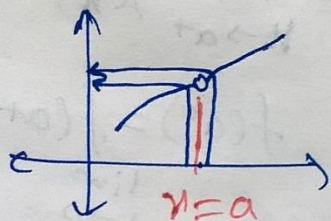
$$= \lim_{x \rightarrow a-h} f(x) = f(a^-) = \lim_{h \rightarrow 0} f(a-h)$$

★ limit of a function at ($x=a$)

$$= \lim_{x \rightarrow a} f(x) \rightarrow \boxed{\lim_{x \rightarrow a^+} f(x)} \quad \parallel \quad \boxed{\lim_{x \rightarrow a^-} f(x)}$$

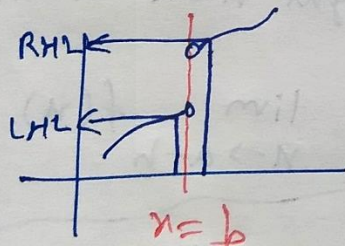
★ If $LHL = RHL$ then
limit exists. = finite

★ If $LHL \neq RHL$ then
 limit does not exist



$LHL = RHL$

Limit exists



$LHL \neq RHL$

limit does not exist

Values $\rightarrow f(x) = 0$

Normal

$\begin{pmatrix} 5, 7 \\ 3, 0 \end{pmatrix}$

OK

$\longleftrightarrow$
 Not Defined (∞)

Does not exist $\sqrt{-1} = i$

OK

Indeter-
-minate
Forms

Dangerous

★ Chapter-13
Limit

Indeterminate Forms

$\frac{0}{0}, \frac{\infty}{\infty}, \infty \cdot 0, \infty - \infty, 0^0, \infty^0, \infty^\infty$

Not exact

$(\text{exact } 1)^\infty = 1$

e.g. $\frac{0}{0}$

Find the value of
function $f(x) = \frac{x^2 - 9}{x - 3}$ at $x = 3$.

Domain $x - 3 \neq 0$
 $x \neq 3$

We can not find the value
of $f(x)$ at $x = 3$ but we
can find value of $f(x)$ in
the neighborhood of $x = 3$.

→ limit at $x = 3$

$$f(3) = \frac{(3)^2 - 9}{(3) - 3} = \frac{9 - 9}{3 - 3} = \frac{0}{0}$$

Indeterminate
Form.

$$f(x) = \frac{x^2 - 9}{x - 3}$$

By Factorisation

$$f(x) = \frac{(x-3)(x+3)}{(x-3)}$$

$x = 3$

$$f(x) = x + 3$$

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} \quad \frac{0}{0}$$

$$= \lim_{x \rightarrow 3} (x + 3)$$

$$= 3 + 3$$

$$= 6$$

$$RHL = 3^+ + 3 = 6^+$$

$$LHL = 3^- + 3 = 6^-$$

Algebra of limits

$$(i) \lim_{x \rightarrow a} (f(x) \pm g(x)) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$(ii) \lim_{x \rightarrow a} (f(x) \times g(x)) = \left(\lim_{x \rightarrow a} f(x) \right) \cdot \left(\lim_{x \rightarrow a} g(x) \right)$$

$$(iii) \lim_{x \rightarrow a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$$

$$(iv) \lim_{x \rightarrow a} (K \cdot f(x)) \longrightarrow K \cdot \lim_{x \rightarrow a} f(x)$$

$$(v) \lim_{x \rightarrow a} (f(x))^{g(x)} = \left(\lim_{x \rightarrow a} f(x) \right)^{\left(\lim_{x \rightarrow a} g(x) \right)}$$

Limit for Polynomial Functions. = Continuous

$$f(x) = 3x + 1$$



→ do not calculate limits Differently LHL, RHL

→ For limit, Directly put the value of x .

e.g. $f(x) = 3x + 1$

limit at $x = 2$ = ?

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} (3x + 1)$$

$$= \cancel{3(2)} + 1 = 7 \checkmark$$

Limit for Rational Fⁿ. = $\frac{f(x)}{g(x)}$ $\left(\frac{0}{0}\right)$

e.g. $\frac{2x+3}{x-1}$, $\frac{x^2+3x+5}{x^3-9}$

→ By Factorisation

→ By Standard form

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$$

Proof: of $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$

LHS = $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$ (Factorise) $\left(\frac{0}{0}\right)$

= $\lim_{x \rightarrow a} \frac{(x-a)(x^{n-1} + x^{n-2} \cdot a + x^{n-3} \cdot a^2 + \dots + x^0 \cdot a^{n-1})}{(x-a)}$

= $\frac{a^{n-1}}{1} + \frac{a^{n-2} \cdot a}{1} + \frac{a^{n-3} \cdot a^2}{1} + \dots + \frac{a^0 \cdot a^{n-1}}{1}$

= $\frac{a^{n-1}}{1} + \frac{a^{n-1}}{1} + \frac{a^{n-1}}{1} + \dots + \frac{a^{n-1}}{1}$
 $\underbrace{\hspace{10em}}_{n \text{ times.}}$

= $n \cdot a^{n-1}$

e.g. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = n \cdot a^{n-1} = 2 \cdot 3^{2-1} = 2 \times 3^1 = 6$
 $n=2$ $a=3$

Sandwich Theorem :



Always

$$g(n) \leq \underline{f(n)} \leq h(n)$$

$$\Rightarrow \lim_{n \rightarrow a} g(n) \leq \lim_{n \rightarrow a} f(n) \leq \lim_{n \rightarrow a} h(n)$$

Tough

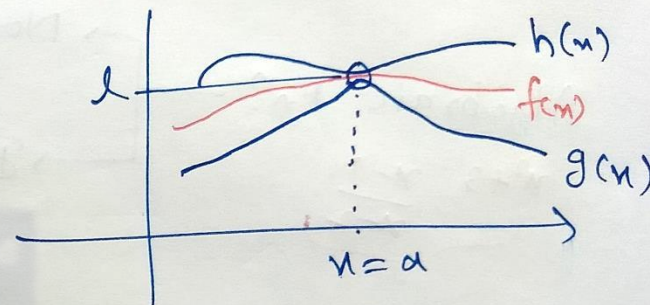


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Easy

Easy



$$\theta \rightarrow 0 \quad \sin \theta \rightarrow \theta$$

By Sandwich theorem Important Result

$$\frac{\sin 0}{0} = \frac{0}{0}$$

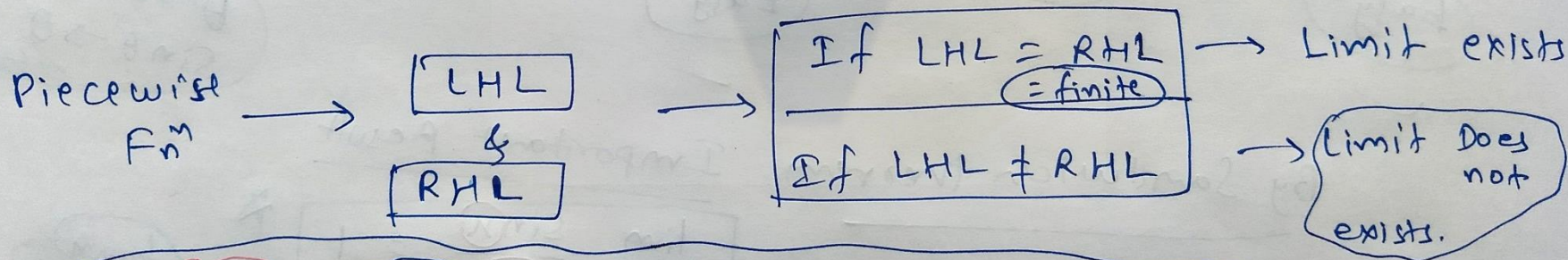
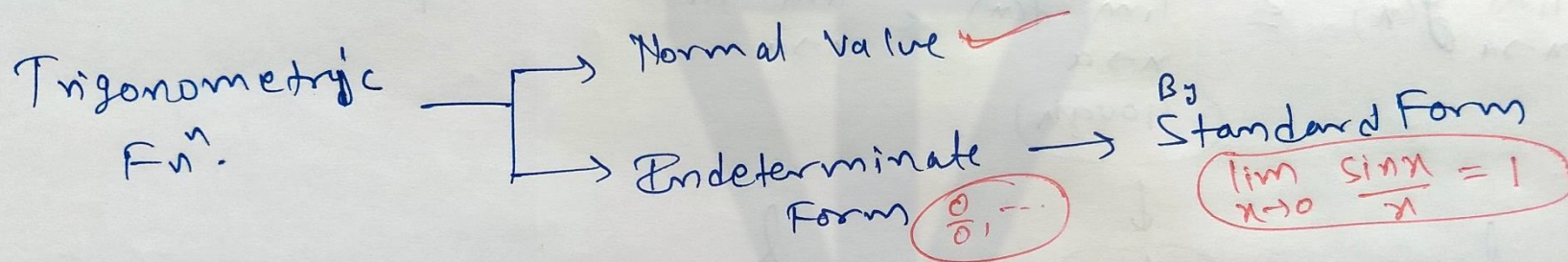
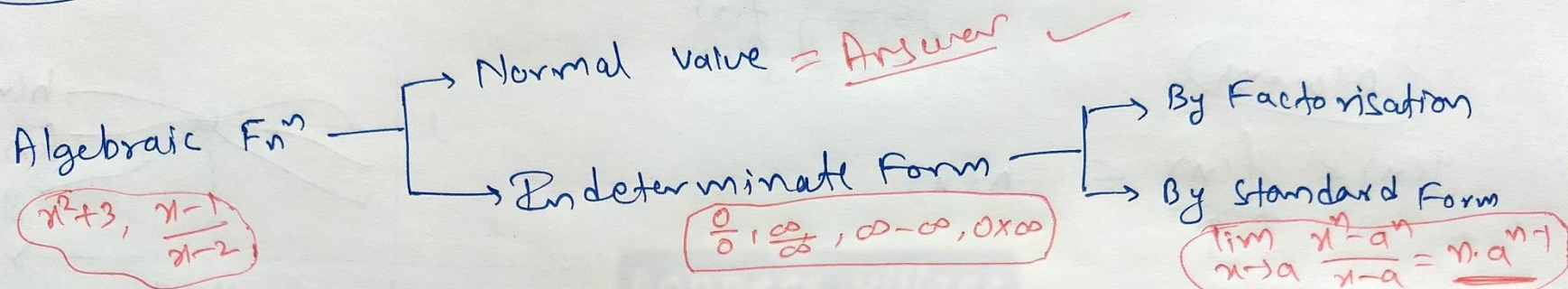
$$\lim_{n \rightarrow 0} \frac{\sin n}{n} = 1$$

$$\lim_{f(n) \rightarrow 0} \frac{\sin f(n)}{f(n)} = 1$$

~~tan~~

$$\lim_{n \rightarrow 0} \cos n = 1$$

Final - Conclusion:



$$f(x) = \begin{cases} x+3, & x \geq 5 \text{ RHL} \\ x-1, & x < 5 \text{ LHL} \end{cases}$$

$\lim_{x \rightarrow 5} f(x)$

LHL = $f(5^-) = 4$

RHL = $f(5^+) = 8$

$\neq$

Exercise-13.1 (Limits & Derivatives)

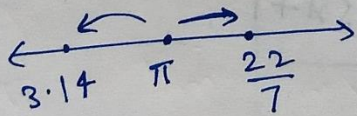
Function $y = f(x)$
(limit at $x = a$)
 $\lim_{x \rightarrow a}$

Put $x = a$
in $y = f(x)$
(Directly)

→ Normal No. $(2, 3, -1, \frac{1}{2}, 0, \infty, \dots) = \text{Answer}$
→ Indeterminate Form $(\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \cdot \infty)$

ways
• Factorisation
• Standard forms
→ $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$
→ $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

Q.1 $\lim_{x \rightarrow 3} x + 3$
(Directly put $x = 3$)
 $= 3 + 3$
 $= 6$ ✓

Q.2 $\lim_{x \rightarrow \pi} (x - \frac{22}{7})$
(Put $x = \pi$ Directly)
 $= \pi - \frac{22}{7}$ ✓


Q.3 $\lim_{x \rightarrow 1} \pi x^2$
put $x = 1$
 $= \pi(1)^2$
 $= \pi$ ✓

Q.4 $\lim_{x \rightarrow 4} \frac{4x + 3}{x - 2}$
($x = 4$ put.)
 $= \frac{16 + 3}{4 - 2} = \frac{19}{2}$ ✓

Q.5 $\lim_{x \rightarrow -1} \frac{x^{10} + x^5 + 1}{x - 1}$

Put $x = -1$

$$= \frac{(-1)^{10} + (-1)^5 + 1}{(-1) - 1}$$

$$= \frac{1 - 1 + 1}{-1 - 1} = \frac{1}{-2} \checkmark$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{(x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1) - 1}{x}$$

$$= \lim_{x \rightarrow 0} (x^4 + 5x^3 + 10x^2 + 10x + 5)$$

$$= 0 + 0 + 0 + 0 + 5$$

$$= 5$$

Put $x = 5$

Q.6 $\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$

Put $x = 0$ $\frac{1^5 - 1}{0} = \frac{0}{0} = \text{Indeterminate Form}$

$(x+1)^5 \rightarrow \text{Expand}$

$$(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

$\downarrow$
 $(x+1)^2 \cdot (x+1)^3$ $\textcircled{5C0}$

$$\boxed{Q.7} \quad \lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$$

Put $x=2$
Direct $\frac{12 - 2 - 10}{4 - 4} = \frac{0}{0}$ Form

(Factorisation)

$$= \lim_{x \rightarrow 2} \frac{3x^2 - 6x + 5x - 10}{(x-2)(x+2)}$$

$$= \lim_{x \rightarrow 2} \frac{3x(x-2) + 5(x-2)}{(x-2)(x+2)}$$

$$= \lim_{x \rightarrow 2} \frac{\cancel{(x-2)}(3x+5)}{\cancel{(x-2)}(x+2)}$$

Put $x=2$

$$= \frac{6+5}{2+2} = \frac{11}{4} \checkmark$$

$$\boxed{Q.8} \quad \lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$$

$$(81 = 3^4)$$

Put $x=3$ Directly $\frac{3^4 - 81}{2(3)^2 - 5(3) - 3} = \frac{0}{0}$
 (Form)

$$= \lim_{x \rightarrow 3} \frac{(x^2)^2 - (3^2)^2}{2x^2 - 6x + x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{(x^2 - 3^2)(x^2 + 3^2)}{2x(x-3) + 1(x-3)}$$

$$= \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x+3)(x^2+9)}{\cancel{(x-3)}(2x+1)}$$

$$= \lim_{x \rightarrow 3} \frac{(x+3)(x^2+9)}{2x+1}$$

$$= \frac{6 \times 18}{7} = \frac{108}{7} \checkmark$$

Q.9 $\lim_{n \rightarrow 0} \frac{an+b}{cn+1}$

Put $n=0$

$$= \frac{0+b}{0+1} = b \checkmark$$

Q.10 $\lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$

Put $z=1$ $\frac{1-1}{1-1} = \frac{0}{0}$ Form

Standard Form

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$$

$$\lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

$(z-1)$ multiply & divide

$$= \lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1} \times \frac{z-1}{z-1}$$

$$= \lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1^{\frac{1}{3}}}{z-1} \times \left(\frac{z-1}{z^{\frac{1}{6}} - 1} \right)$$

$$= \left(\frac{1}{3} \cdot (1)^{\frac{1}{3}-1} \right) \times \left(\frac{1}{\frac{1}{6} \cdot 1^{\frac{1}{6}-1}} \right)$$

$$= \frac{1}{3} \times (6)$$

$$= 2 \checkmark$$

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Exercise 13.1

Class -11

Q.11 $\lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}$

$a + b + c \neq 0$

$x=1$ put
 $\Rightarrow \frac{a+b+c}{c+b+a} = 1$

Q.12 $\lim_{x \rightarrow -2} \left(\frac{\frac{1}{x} + \frac{1}{2}}{x+2} \right)$ Put $x=-2$
 $\left(\frac{0}{0} \right)$

$= \lim_{x \rightarrow -2} \frac{\left(\frac{2+x}{2x} \right)}{(x+2)}$

$= \lim_{x \rightarrow -2} \left(\frac{1}{2x} \right)$

$= \left(\frac{1}{-4} \right)$

Q.13 $\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$

Put $x=0$

$\frac{\sin 0}{0} = \frac{0}{0}$ Form

Standard Form

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$= \lim_{x \rightarrow 0} \frac{\sin ax}{bx} \times \frac{a}{a}$

$= \left(\lim_{x \rightarrow 0} \frac{\sin ax}{ax} \right) \times \frac{a}{b} = 1 \times \frac{a}{b} = \frac{a}{b}$

Q.14 $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$, $a, b \neq 0$

$= \lim_{x \rightarrow 0} \left(\frac{\sin ax}{ax} \right) \times \frac{bx}{\sin bx} \times \frac{a}{b}$

$= 1 \times 1 \times \frac{a}{b}$

$= \frac{a}{b}$

Q.15 $\lim_{x \rightarrow \pi} \frac{\sin(\pi-x)}{\pi(\pi-x)}$

By Standard Form

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$= \frac{1}{\pi} \times 1 = \frac{1}{\pi}$$

$$\frac{\sin(\pi-\pi)}{\pi \cdot (\pi-\pi)} = \frac{\sin 0}{\pi \times 0} = \frac{0}{0}$$

Q.16 $\lim_{x \rightarrow 0} \frac{\cos x}{\pi-x}$

Put $x=0$ Direct

$$= \frac{\cos 0}{\pi-0} = \frac{1}{\pi}$$

Q.17

$$\lim_{x \rightarrow 0} \frac{(\cos 2x) - 1}{\cos x - 1}$$

Put $x=0$

$$\frac{\cos 0 - 1}{\cos 0 - 1} = \frac{1-1}{1-1} = \frac{0}{0}$$

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\cos x = 1 - 2 \sin^2 \frac{x}{2}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - 2 \sin^2 x) - 1}{(1 - 2 \sin^2 \frac{x}{2}) - 1} = \lim_{x \rightarrow 0} \frac{-2 \sin^2 x}{-2 \sin^2 \frac{x}{2}}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{\frac{x}{2} \times 2}{\sin \frac{x}{2}} \right)^2$$

$$= (1 \times 1 \times 2)^2$$

$$= 4$$

Q.18 $\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$

Std Form
 $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
 0 Form

$$= \lim_{x \rightarrow 0} \frac{\left(\frac{ax + x \cos x}{x} \right)}{\left(b \frac{\sin x}{x} \right)} \rightarrow 1$$

$$= \lim_{x \rightarrow 0} \frac{a + \cos x}{b x \left(\frac{\sin x}{x} \right)}$$

$$= \frac{a + \cos 0}{b \times 1} = \frac{a+1}{b}$$

$\frac{1 - \cos^2 x}{\sin^2 x} = \sin^2 x$

Q.19 $\lim_{x \rightarrow 0} x \sec x$

$x=0$ put
 $\sec 0 = 1$
 $= 0 \times 1 = 0$

Q.21

$$\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$$

put $x=0$
 $= \operatorname{cosec} 0 - \cot 0$
 $= \infty - \infty$
 Indeterminate

$$= \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{\sin x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{(1 - \cos x)}{\sin x} \cdot \frac{(1 + \cos x)}{(1 + \cos x)}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\sin x \cdot (1 + \cos x)}$$

Q.20 $\lim_{x \rightarrow 0} \frac{\sin ax + bx}{a + \sin bx}$ $\left(\frac{0}{0} \right)$

$$= \lim_{x \rightarrow 0} \frac{\left(\frac{\sin ax + bx}{x} \right)}{\left(\frac{a + \sin bx}{x} \right)}$$

$$= \lim_{x \rightarrow 0} \frac{\left(\frac{\sin ax}{ax} \right) \times a + b}{a + \left(\frac{\sin bx}{bx} \right) \times b}$$

$$= \frac{1 \times a + b}{a + 1 \times b} = \frac{a+b}{a+b} = 1$$

$$= \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin x \cdot (1 + \cos x)}$$

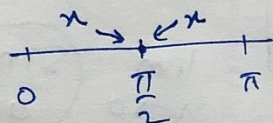
$$= \lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x}$$

$$= \frac{\sin 0}{1 + \cos 0} = \frac{0}{1+1} = \frac{0}{2} = 0$$

Q.22 $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

Put $x = \frac{\pi}{2}$ $\frac{\tan 2(\frac{\pi}{2})}{\frac{\pi}{2} - \frac{\pi}{2}} = \frac{0}{0}$ Form

$\lim_{x \rightarrow \frac{\pi}{2}}$



$x = \frac{\pi}{2} + h$

Substitute $\uparrow$

$h \rightarrow 0$ $h \approx 0$
 $h = 0.000000001$

$\lim_{(\frac{\pi}{2} + h) \rightarrow \frac{\pi}{2}} \left(\frac{\tan 2(\frac{\pi}{2} + h)}{(\frac{\pi}{2} + h) - \frac{\pi}{2}} \right)$

$= \lim_{h \rightarrow 0} \frac{\tan(\pi + 2h)}{h}$ $\tan(\pi + \theta) = \tan \theta$

$= \lim_{h \rightarrow 0} \frac{\tan 2h}{h}$

Std Form

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$= \lim_{h \rightarrow 0} \frac{\tan 2h}{h}$

$= \lim_{h \rightarrow 0} 2 \cdot \frac{\sin 2h}{2h \cdot \cos 2h}$

$= \frac{2 \times 1}{\cos 0} = \frac{2}{1}$

$= 2$

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Exercise - 13.1 Limits & Derivatives

Piecewise Functions

$$f(x) = \begin{cases} x^2 + 3, & x \geq a \\ 2x - 1, & x < a \end{cases}$$

Critical Point $\swarrow$

limit at critical point $\swarrow$ limit at non-critical point

LHL $\swarrow$ $x \geq a$ $\swarrow$ a^- $\swarrow$ $x < a$ $\swarrow$ $2a - 1$

RHL $\swarrow$ $x > a$ $\swarrow$ a^+ $\swarrow$ $x > a$ $\swarrow$ $a^2 + 3$

LHL = RHL $\checkmark$
LHL $\neq$ RHL $\times$

$x = a$
Directly put

$x = 0$
Here LHL = RHL $= 3$
 $\lim_{x \rightarrow 0} f(x) = 3$

Q.23

$$f(x) = \begin{cases} 2x + 3, & x \leq 0 \\ 3(x + 1), & x > 0 \end{cases}$$

Critical point $x = 0$

$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} 3(x + 1)$

Directly put $= 3(1 + 1) = 3 \times 2 = 6 \checkmark$

$\lim_{x \rightarrow 0} f(x)$ Critical point $\swarrow$ Left $\searrow$ Right $\frac{0/0^+}{0}$

LHL $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (2x + 3) = 3$

RHL $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 3(x + 1) = 3(0 + 1) = 3$

Q.24

$$f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{cases}$$

Critical point = 1

$$\lim_{x \rightarrow 1} f(x)$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 1^-} f(x) \\ &= \lim_{x \rightarrow 1^-} (x^2 - 1) \end{aligned}$$

$$= (1)^2 - 1$$

$$= 1 - 1$$

$$= 0 = \text{LHL}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 1^+} f(x) \\ &= \lim_{x \rightarrow 1^+} (-x^2 - 1) \end{aligned}$$

$$= -(1)^2 - 1$$

$$= -1 - 1$$

$$= -2 = \text{RHL}$$

$$\text{LHL} \neq \text{RHL}$$

$\therefore \lim_{x \rightarrow 1} f(x)$ does not exist

Q.25

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

~~Q.25~~

$$\lim_{x \rightarrow 0} f(x) \quad \begin{matrix} 0^- & 0^+ = 0+h \\ \leftarrow & \rightarrow \end{matrix} \quad \text{Critical point} = 0$$



$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) \\ &= \lim_{x \rightarrow 0^-} \frac{|x|}{x} \end{aligned}$$

$$\begin{matrix} 0^- = 0-h \\ h > 0 \\ h \rightarrow 0 \end{matrix}$$

$$= \lim_{h \rightarrow 0} \frac{|-h|}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{h}{-h}$$

$$= -1 = \text{LHL}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 0^+} f(x) \\ &= \lim_{x \rightarrow 0^+} \frac{|x|}{x} \end{aligned}$$

$$= \lim_{x \rightarrow 0^+} \frac{|x|}{x}$$

$$0^+ = 0+h$$

$$= \lim_{h \rightarrow 0} \frac{|0+h|}{0+h}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h}$$

$$= 1 = \text{RHL}$$



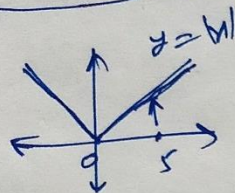
$$\text{LHL} \neq \text{RHL}$$

$\therefore$ Limit $\rightarrow$ Not exist

Q.27

$$f(x) = |x| - 5$$

$$\lim_{x \rightarrow 5} f(x)$$



$|x|$ changes its nature at $x=0$

$$\begin{aligned} \lim_{x \rightarrow 5} f(x) &= |5| - 5 \\ &= 5 - 5 \\ &= 0 \end{aligned}$$

Q.28

$$f(x) = \begin{cases} a+bx, & x < 1 \\ 4, & x = 1 \\ b-ax, & x > 1 \end{cases}$$

Given $\lim_{x \rightarrow 1} f(x) = f(1) = 4$

$a, b = ?$

limit neighborhood

Exact

LHL = RHL

$LHL = RHL = 4$

$$\Rightarrow \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = 4$$

$$\Rightarrow \lim_{x \rightarrow 1^-} (a+bx) = \lim_{x \rightarrow 1^+} (b-ax) = 4$$

$$\Rightarrow \begin{matrix} a+b & = & b-a & = & 4 \end{matrix}$$

$a=0$
 $b=4$

$$\begin{aligned} \Rightarrow a+b &= 4 \\ \Rightarrow b &= 4 \end{aligned} \quad \left\{ \begin{aligned} a+b &= b-a \\ \Rightarrow 2a &= 0 \Rightarrow a=0 \end{aligned} \right.$$

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Exercise 13.1 Limits & Derivatives

Q.29 $f(x) = (x-a_1)(x-a_2)\dots(x-a_n)$

$$\begin{aligned}\checkmark \lim_{x \rightarrow a_1} f(x) &= \lim_{x \rightarrow a_1} (x-a_1)(x-a_2)\dots(x-a_n) \\ &= (a_1-a_1)(a_1-a_2)\dots(a_1-a_n) = 0\end{aligned}$$

$$\begin{aligned}\checkmark \lim_{x \rightarrow a} f(x) &= \lim_{x \rightarrow a} (x-a_1)(x-a_2)\dots(x-a_n) \\ &= (a-a_1)(a-a_2)\dots(a-a_n)\end{aligned}$$

Q.30

$$f(x) = \begin{cases} |x|+1, & x < 0 \\ 0, & x = 0 \\ |x|-1, & x > 0 \end{cases}$$

For what value(s) of a does
 $\lim_{x \rightarrow a} f(x)$ exist?

$\therefore$ Here $|x|+1$ & $|x|-1$
are algebraic functions,
therefore these functions
will be continuous in
their domain

~~$\Rightarrow$ They will be~~

$\Rightarrow$ limit will exist
at every point in
their domain.

But

Critical point $x=0$

(We have to check only
on this point)

$$\lim_{x \rightarrow 0} f(x)$$

LHL

RHL

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} (|x| + 1)$$

$$= |0| + 1$$

$$= 0 + 1$$

$$= 1$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x)$$

$$= \lim_{x \rightarrow 0^+} (|x| - 1)$$

$$= |0| - 1$$

$$= -1$$

At $x=0$, $\text{LHL} \neq \text{RHL}$

$\therefore$ limit does not exist at $x=0$.

$$\left(\frac{0}{0} \quad \frac{\infty}{\infty} \quad \frac{0 \cdot \infty}{\infty \cdot \infty} \right)$$

X

$$\lim_{x \rightarrow 0^+} \frac{1}{x}$$

$\therefore \lim_{x \rightarrow a} f(x)$
exists at
every $a \in \mathbb{R} - \{0\}$

~~Q.30~~ (Q.31) $\lim_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} = \pi$

$$\lim_{x \rightarrow 1} f(x) = ?$$

$$\Rightarrow \frac{\lim_{x \rightarrow 1} f(x) - 2}{\lim_{x \rightarrow 1} (x^2) - 1} = \pi$$

$$\frac{\lim_{x \rightarrow 1} f(x) - 2}{1 - 1} = \pi$$

$$\frac{\lim_{x \rightarrow 1} f(x) - 2}{0} = \pi$$

$$\Rightarrow \text{If } \lim_{x \rightarrow 1} f(x) \neq 2$$

$$0 \neq \frac{\lim_{x \rightarrow 1} f(x) - 2}{\lim_{x \rightarrow 1} x^2 - 1} = \infty \neq \pi$$

Impossible

$\therefore \lim_{x \rightarrow 1} f(x)$ should be equal to '2'

$$\frac{0}{0}$$

Q.32

$$f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1 \\ nx^3 + m, & x > 1 \end{cases}$$

Integers
↓ ↓
m, n = ?

$\lim_{x \rightarrow 0} f(x)$
exist

$\lim_{x \rightarrow 1} f(x)$
exist

$\therefore \lim_{x \rightarrow 0} f(x)$ exists.

LHL = RHL

$$\Rightarrow \lim_{x \rightarrow 0^-} (f(x)) = \lim_{x \rightarrow 0^+} (f(x))$$

$$\Rightarrow \lim_{x \rightarrow 0^-} (mx^2 + n) = \lim_{x \rightarrow 0^+} (nx + m)$$

$$\Rightarrow m(0)^2 + n = n(0) + m$$

$$\Rightarrow n = m$$

$\lim_{x \rightarrow 1} f(x)$ exists

LHL = RHL

$$\Rightarrow \lim_{x \rightarrow 1^-} (f(x)) = \lim_{x \rightarrow 1^+} (f(x))$$

$$\Rightarrow \lim_{x \rightarrow 1^-} (nx + m) = \lim_{x \rightarrow 1^+} (nx^3 + m)$$

$$\Rightarrow n(1) + m = n(1) + m$$

$$\Rightarrow \underline{n + m} = \underline{n + m}$$

⊗ This is always true for every value of m & n.
↓
integral

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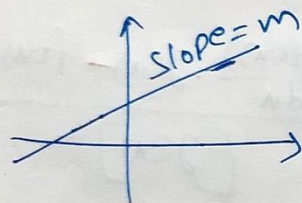
Class-11-Maths Chapter-13

Derivatives → Rate of change
(Slope)

→ x

Straight line

$$y = mx + c$$

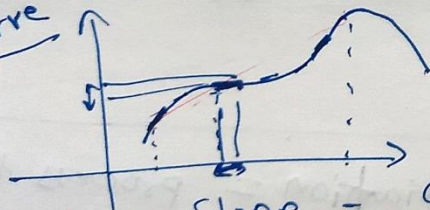


$P(x_1, y_1)$ $Q(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

$$\text{Slope} = m = \frac{\text{change in 'y'}}{\text{change in 'x'}} = \frac{\text{change in output}}{\text{change in input}}$$

For Curve

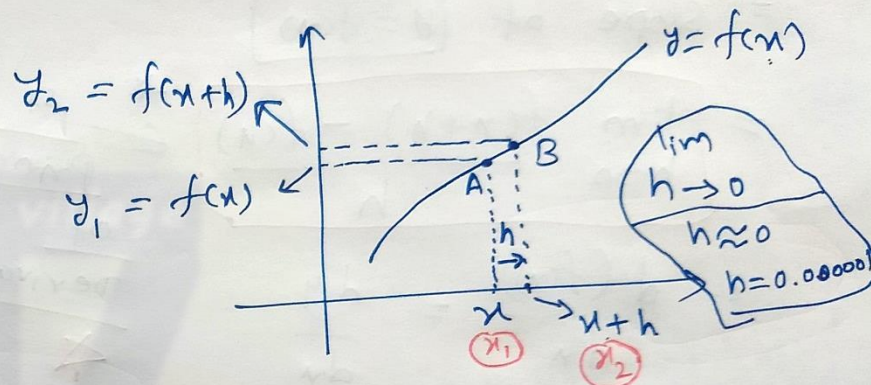


instantaneous slope

↓
Derivative

$$= \frac{\text{change in output}}{\text{very very change in input small}}$$

Derivative of a Function $y = f(x)$



Derivative of $y = f(x)$

= rate of change of $y = f(x)$

$$= \frac{\text{change in output}}{\text{v.v. small change in input}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

★ Derivative of $[y = f(x)]$

= Slope of $[y = f(x)]$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

★ 1st Principle of Derivative ★

$$= \frac{d(f(x))}{dx} = \frac{dy}{dx}$$

$$= f'(x) = y'$$

★ Derivative (slope) at Particular Point $x=a$

$$= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \left. \frac{d(f(x))}{dx} \right|_{x=a} = \left. \frac{dy}{dx} \right|_{x=a}$$

$$= f'(a)$$

Algebra of Derivatives

$$\textcircled{1} \frac{d[f(x) + g(x)]}{dx} = \frac{d f(x)}{dx} + \frac{d(g(x))}{dx}$$

$$\textcircled{2} \frac{d[k \cdot f(x)]}{dx} = k \frac{d(f(x))}{dx}$$

$$\textcircled{3} \frac{d\left[\frac{f(x)}{g(x)}\right]}{dx} = \frac{\frac{d(f(x))}{dx} \cdot g(x) - f(x) \cdot \frac{d(g(x))}{dx}}{[g(x)]^2}$$

$$\textcircled{4} \frac{d(f(x) \cdot g(x))}{dx} = \frac{d f(x)}{dx} \cdot g(x) + f(x) \cdot \frac{d(g(x))}{dx}$$

Differentiation = process to find derivative

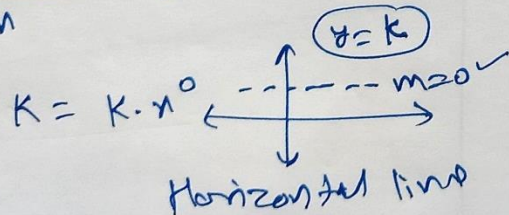
$$\star (u.v)' = u'.v + u.v'$$

$$\star \left(\frac{u}{v}\right)' = \frac{u'.v - u.v'}{v^2}$$

Important Derivatives

$$\left\{ \begin{array}{l} \frac{d(x^n)}{dx} = n \cdot x^{n-1} \\ \frac{d(\sin x)}{dx} = \cos x \\ \frac{d(\cos x)}{dx} = -\sin x \end{array} \right.$$

$$\frac{d(\text{Constant})}{dx} = 0 \checkmark$$



Derivative of $y = f(x) = x^n$ using 1st Principle

$$\begin{aligned} \frac{d(x^n)}{dx} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} \end{aligned}$$

$$(x+h)^n = nC_0 \cdot x^n \cdot h^0 + nC_1 \cdot x^{n-1} \cdot h^1 + \dots + nC_n \cdot x^0 \cdot h^n$$

Binomial Theorem

$$nC_r = \frac{n!}{(n-r)!r!}$$

$$nC_0 = 1 = nC_n$$

$$nC_1 = n = nC_{n-1}$$

$$= x^n + n \cdot x^{n-1} \cdot h + \dots + h^n$$

$$\frac{d(x^n)}{dx} = \lim_{h \rightarrow 0} \frac{[x^n + n \cdot x^{n-1} \cdot h + \dots + h^n] - x^n}{h}$$

$$= \lim_{h \rightarrow 0} \left(n \cdot x^{n-1} + \frac{n(n-1)}{2} \cdot x^{n-2} \cdot h + \dots + h^{n-1} \right)$$

$$= n \cdot x^{n-1} \checkmark$$

Derivative of $y = \sin x = f(x)$

(using first principle)

$$\frac{d(\sin x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$\boxed{\sin A - \sin B = 2 \sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right)}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin\left(\frac{x+h-x}{2}\right) \cdot \cos\left(\frac{x+h+x}{2}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right) \cdot \cos\left(x + \frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$= 1 \times \cos(x+0)$$

$$= \cos x$$

$$\boxed{\frac{d(\sin x)}{dx} = \cos x}$$

QED

eg. find Derivative of ' $x^2 \cdot \sin x$ '
(u) x (v)

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

$$u = x^2$$

$$v = \sin x$$

$$u = x^2 \rightarrow u' = \frac{d(x^2)}{dx} = 2 \cdot x^{2-1} \\ = 2 \cdot x^1 \\ = 2x$$

$$v = \sin x \rightarrow v' = \frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(x^2 \cdot \sin x)}{dx} = \underline{(2x) \cdot \sin x + x^2 \cdot \cos x}$$

eg. find derivative of ' $\tan x$ '

$$\tan x = \frac{\sin x}{\cos x}$$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$\frac{d(\tan x)}{dx} = \frac{d\left(\frac{\sin x}{\cos x}\right)}{dx}$$

$$= \frac{(\sin x)' \cos x - (\sin x) \cdot (\cos x)'}{\cos^2 x}$$

$$= \frac{\cos x \cdot \cos x - (\sin x) \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x \checkmark$$

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Exercise 13.2 Derivatives

[Q.1] Derivative of $x^2 - 2 = f(x)$
at $x = 10$.

By first principle of Derivative

$$\begin{aligned}\left. \frac{d(x^2 - 2)}{dx} \right|_{x=10} &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{f(10+h) - f(10)}{h} \\&= \lim_{h \rightarrow 0} \frac{[(10+h)^2 - 2] - [(10)^2 - 2]}{h} \\&= \lim_{h \rightarrow 0} \frac{100 + h^2 + 20h - \cancel{2} - 100 + \cancel{2}}{h}\end{aligned}$$

$$= \lim_{h \rightarrow 0} (h + 20)$$

$$= 0 + 20 = 20$$

$$\frac{d(x^2 - 2)}{dx} = (2x^{2-1} - 0) = 2x$$

$x = 10 \rightarrow 20$

Shortcut

[Q.2] Derivative of $99x$ at $x = 100$
 $f(x)$

$$\begin{aligned}\left. \frac{d(99x)}{dx} \right|_{x=100} &= \lim_{h \rightarrow 0} \frac{f(100+h) - f(100)}{h} \\&= \lim_{h \rightarrow 0} \frac{99(100+h) - 99 \times 100}{h} \\&= \lim_{h \rightarrow 0} \frac{\cancel{99 \times 100} + 99xh - \cancel{99 \times 100}}{h} \\&= 99\end{aligned}$$

Q.3 Derivative of ' x ' = $f(x)$
at $x=1$.

$$\left. \frac{d(x)}{dx} \right|_{x=1} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h) - (x)}{h}$$

$$\rightarrow = 1$$

Q.4 "Derivative Using
First Principle".

(i) $x^3 - 27 = f(x)$

$$\frac{d(f(x))}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(x+h)^3 - 27] - [x^3 - 27]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^3} + h^3 + 3x^2h + 3xh^2 - \cancel{27} - \cancel{x^3} + \cancel{27}}{h}$$

$$\boxed{(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2}$$
$$(x+h)^3 = \quad - \quad - \quad -$$

$$= \lim_{h \rightarrow 0} \frac{h^3 + 3x^2h + 3xh^2}{h} \quad \left(\frac{0}{0} \right)$$

$$= \lim_{h \rightarrow 0} (h^2 + 3x^2 + 3xh)$$

$$= 0 + 3x^2 + 0 = 3x^2$$

$$\textcircled{4} \text{ (ii) } (x-1)(x-2) = f(x)$$

$$\Rightarrow f(x) = x^2 - 3x + 2$$

$$\frac{d(f(x))}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

(1st principle) ↗

$$= \lim_{h \rightarrow 0} \frac{[(x+h)^2 - 3(x+h) + 2] - [x^2 - 3x + 2]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{3x} - 3h + \cancel{2} - \cancel{x^2} + \cancel{3x} - \cancel{2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2hx + h^2 - 3h}{h}$$

$$= \lim_{h \rightarrow 0} (2x + \underset{\uparrow}{h} - 3) = \underline{\underline{2x - 3 \text{ Ans.}}}$$

$$\textcircled{4} \text{ (iii) } \frac{1}{x^2} = f(x)$$

$$\frac{d(f(x))}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h \rightarrow 0} \left\{ \frac{\cancel{x^2} - (x+h)^2}{\cancel{x^2} \cdot (x+h)^2} \cdot \frac{h}{1} \right\}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^2} - \cancel{x^2} - h^2 - 2hx}{h \cdot \cancel{x^2} \cdot (x+h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{-\cancel{h}(h+2x)}{\cancel{h} \cdot (x^2) \cdot (x+h)^2}$$

$$= \frac{-2x}{x^2 \cdot x^2} = -\frac{2}{x^3}$$

$$\textcircled{4} \textcircled{\text{IV}} \quad \frac{x+1}{x-1} = f(x)$$

$$\frac{d(f(x))}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x^2 - x + hx - h + x - 1) - (x^2 + x - x - 1)}{h(x+h-1)(x-1)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^2} - \cancel{x} + \cancel{hx} - \cancel{h} + \cancel{x} - 1 - \cancel{x^2} - \cancel{x} + \cancel{x} + 1 - \cancel{hx} - \cancel{h}}{h(x+h-1)(x-1)}$$

$$= \lim_{h \rightarrow 0} \frac{-2h}{h(x+h-1)(x-1)}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{\cancel{(x+h)}(x+h-1)(x-1)}$$

$$= \frac{-2}{(x-1)^2}$$

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Class - 11 - Maths Exercise 13.2

Q.5 Prove $f'(1) = 100 \cdot f'(0)$ $(1=x^0)$

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$$

$$f'(x) = ? \quad \left[\frac{d(x^n)}{dx} = n \cdot x^{n-1} \right] \quad \frac{d(\text{Const})}{dx} = 0$$

$$f'(x) = \frac{100 \cdot x^{99}}{100} + \frac{99 \cdot x^{98}}{99} + \dots + \frac{2 \cdot x^1}{2} + 1 \cdot x^0 + 0$$

$$f'(x) = x^{99} + x^{98} + \dots + x + 1$$

$$f'(1) = \underbrace{1 + 1 + \dots + 1 + 1}_{100 \text{ times}} = 100$$

$$f'(0) = 0 + 0 + \dots + 0 + 1 = 1$$

$$\underline{f'(1)} = 100 = 100 \times 1 = \underline{100 \times f'(0)}$$

Q.6 $a = \text{some fixed real no.}$

$$f(x) = x^n + ax^{n-1} + a^2x^{n-2} + \dots + a^{n-1}x + a^n$$

$$\frac{d(f(x))}{dx} = \frac{d(x^n + \dots + a^n)}{dx}$$

$$= n \cdot x^{n-1} + (n-1) \cdot a \cdot x^{n-2} + (n-2) \cdot a^2 \cdot x^{n-3} + \dots + a^{n-1} \cdot 1 + 0$$

Q.7 $a, b \rightarrow \text{constants}$

(i) $(x-a)(x-b) = f(x)$

$$\Rightarrow f(x) = x^2 - ax - bx + ab$$

$$f(x) = x^2 - x(a+b) + ab$$

$$\frac{d(f(x))}{dx} = 2x - 1 \cdot (a+b) + 0$$

$$= 2x - a - b \checkmark$$

$$(ii) f(x) = (ax^2 + b)^2$$

$$f(x) = (ax^2 + b) \cdot (ax^2 + b)$$

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

$$\frac{d(f(x))}{dx} = (a \cdot 2x' + 0) \cdot (ax^2 + b)$$

$$+ (ax^2 + b) \cdot (a \cdot 2x' + 0)$$

$$= (ax^2 + b) \cdot \{2ax + 2ax\}$$

$$= 4ax \cdot (ax^2 + b)$$

$$(iii) \frac{u}{v} = \frac{x-a}{x-b} = f(x)$$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$\frac{d(f(x))}{dx} = f'(x) = \frac{(1-0) \cdot (x-b) - (x-a) \cdot (1-0)}{(x-b)^2}$$

$$f'(x) = \frac{(x-b) - (x-a)}{(x-b)^2}$$

$$f'(x) = \frac{x-b - x+a}{(x-b)^2}$$

$$f'(x) = \frac{a-b}{(x-b)^2}$$

Q.8

$a = \text{some constant}$

$$f(x) = \frac{x^n - a^n}{x - a} = \frac{u}{v}$$

$$\begin{aligned} x^n &\rightarrow n \cdot x^{n-1} \\ x^1 &\rightarrow 1 \cdot x^{1-1} = 1 \cdot x^0 = 1 \cdot 1 = 1 \end{aligned}$$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$\frac{d(f(x))}{dx} = f'(x) = \frac{(n \cdot x^{n-1} - 0) \cdot (x - a) - (x^n - a^n) \cdot (1 - 0)}{(x - a)^2}$$

$$= \frac{n \cdot x^{n-1} \cdot (x - a) - (x^n - a^n)}{(x - a)^2}$$

$$= \frac{n \cdot x^n - n \cdot a \cdot x^{n-1} - x^n + a^n}{(x - a)^2}$$

$$= \frac{(n-1)x^n - na x^{n-1} + a^n}{(x - a)^2}$$

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Exercise 13.2 Limits & Derivatives

Q.9 (i) $2x - \frac{3}{4} = f(x)$ $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$

Derivative $f'(x) = \frac{d(2x - \frac{3}{4})}{dx}$

$$f'(x) = \frac{2 \frac{d(x)}{dx}}{dx} - \frac{d(\frac{3}{4})}{dx} \rightarrow 0$$

$$= 2 \times 1 \cdot x^{1-1} - 0$$

$$= 2 \times x^0 = 2$$

(ii) $(5x^3 + 3x - 1)(x - 1) = f(x)$

$$f(x) = 5x^4 - 5x^3 + 3x - 3 - x + 1$$

$$f(x) = 5x^4 - 5x^3 + 2x - 2$$

$$f'(x) = 5 \cdot 4 \cdot x^3 - 5 \cdot 3 \cdot x^2 + 2 - 0$$

$$f'(x) = 20x^3 - 15x^2 + 2$$

(iii) $x^{-3}(5 + 3x) = f(x)$

$$\Rightarrow f(x) = 5x^{-3} + 3x^{-2}$$

Derivative

$$\Rightarrow f'(x) = (-3) \cdot 5x^{-4} + (-2) \cdot 3x^{-3}$$

$$f'(x) = -15x^{-4} - 6x^{-3}$$

$$(iv) f(x) = x^5 \cdot (3 - 6x^{-9})$$

$$f(x) = 3x^5 - 6 \cdot x^{-4}$$

Derivative

$$\Rightarrow f'(x) = 5 \cdot (3x^4) - (-4) \cdot 6 \cdot x^{-5}$$

$$\boxed{f'(x) = 15x^4 + 24x^{-5}}$$

$$(v) x^{-4} \cdot (3 - 4x^{-5}) = f(x)$$

$$f(x) = 3x^{-4} - 4 \cdot x^{-9}$$

$$f'(x) = (-4) \cdot 3x^{-5} - (-9) \cdot 4 \cdot x^{-10}$$

$$f'(x) = -12x^{-5} + 36x^{-10}$$

$$(vi) f(x) = \frac{2}{x+1} - \frac{x^2}{3x-1}$$

$$\boxed{\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}}$$

$$f'(x) = \frac{(0) \cdot (x+1) - 2(1+0)}{(x+1)^2}$$

$$- \frac{(2x)(3x-1) - (x^2) \cdot (3x-0)}{(3x-1)^2}$$

$$\Rightarrow f'(x) = \left(\frac{-2}{(x+1)^2}\right) - \left(\frac{6x^2 - 2x - 3x^2}{(3x-1)^2}\right)$$

$$\Rightarrow f'(x) = \left(\frac{-2}{(x+1)^2}\right) - \left(\frac{3x^2 - 2x}{(3x-1)^2}\right)$$

Q.10 Derivative of $\cos x$
(by first principle)

$$f(x) = \cos x$$

$$\frac{d(f(x))}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h}$$

$$\Rightarrow \frac{d(\cos x)}{dx} = \lim_{h \rightarrow 0} \frac{-2 \sin\left(\frac{x+h+x}{2}\right) \cdot \sin\left(\frac{x+h-x}{2}\right)}{h}$$

$$\boxed{\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \sin\left(x + \frac{h}{2}\right) \cdot \sin \frac{h}{2}}{h}$$

Indeterminate $\left(\frac{0}{0} \text{ form}\right)$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$= \lim_{h \rightarrow 0} \frac{-\sin\left(x + \frac{h}{2}\right) \cdot \sin \frac{h}{2}}{\left(\frac{h}{2}\right)}$$

$$= -\sin(x+0) \times 1$$

$$= -\sin x$$

$$\boxed{\frac{d(\cos x)}{dx} = -\sin x}$$

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Exercise (13.2) Derivatives

Q.11 Find the Derivatives

(i) $f(x) = \sin x \cdot \cos x$ $(u \cdot v)' = u'v + u \cdot v'$

$$\frac{d(f(x))}{dx} = f'(x) = \frac{d(\sin x \cdot \cos x)}{dx}$$

$$= \frac{d(\sin x)}{dx} \cdot \cos x + \sin x \cdot \frac{d(\cos x)}{dx}$$

$$= \cos x \cdot \cos x + \sin x \cdot (-\sin x)$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x$$

(ii) $f(x) = \sec x = \frac{1}{\cos x} = \frac{u}{v}$

$$f'(x) = \frac{d\left(\frac{1}{\cos x}\right)}{dx} \quad \left(\frac{u}{v}\right)' = \frac{u'v - u \cdot v'}{v^2}$$

$$= \frac{(0) \cdot \cos x - (1) \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \sec x \cdot \tan x$$

$$(iii) f(x) = 5 \sec x + 4 \cos x$$

$$f'(x) = \frac{d(5 \sec x + 4 \cos x)}{dx}$$

$$= \frac{d(5 \sec x)}{dx} + \frac{d(4 \cos x)}{dx}$$

$$= 5 \cdot \frac{d(\sec x)}{dx} + 4 \cdot \frac{d(\cos x)}{dx}$$

↓ By (ii) Part

$$= 5 \cdot (\sec x \cdot \tan x) - 4 \sin x$$

$$(iv) f(x) = \operatorname{cosec} x = \frac{1}{\sin x} = \frac{u}{v}$$

$$f'(x) = \frac{(0) \cdot \sin x - (1) \cdot (\cos x)}{\sin^2 x}$$

$$\left(\frac{u}{v}\right)' = \frac{u \cdot v - u \cdot v'}{v^2}$$

$$= \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \cdot \operatorname{cosec} x$$

$$(v) f(x) = 3 \cot x + 5 \operatorname{cosec} x$$

$$f(x) = \frac{3 \cos x}{\sin x} + \frac{5}{\sin x}$$

$$\rightarrow \left(\frac{u}{v}\right)' = \frac{u \cdot v' - u \cdot v'}{v^2}$$

$$f'(x) = \frac{3 \frac{d(\cos x)}{\sin x}}{dx} + 5 \cdot \frac{d\left(\frac{1}{\sin x}\right)}{dx}$$

$$= 3 \cdot \frac{(-\sin x) \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$$

$$+ 5 (-\cot x \cdot \operatorname{cosec} x)$$

$$= -3 (\sin^2 x + \cos^2 x) / \sin^2 x - 5 \operatorname{cosec} x \cdot \cot x$$

$$= -3 \operatorname{cosec}^2 x - 5 \operatorname{cosec} x \cdot \cot x$$

$$(vi) f(x) = 5 \sin x - 6 \cos x + 7$$

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\text{constant})}{dx} = 0$$

$$f'(x) = \frac{d(5 \sin x - 6 \cos x + 7)}{dx}$$

$$= 5 \cdot (\cos x) - 6(-\sin x) + 0$$

$$= 5 \cos x + 6 \sin x$$

✓

$$(vii) f(x) = 2 \tan x - 7 \sec x$$

$$\textcircled{\text{~~1~~}} f(x) = 2 \frac{\sin x}{\cos x} - \frac{7}{\cos x}$$

$$f'(x) = 2 \cdot \frac{d\left(\frac{\sin x}{\cos x}\right)}{dx} - 7 \cdot \frac{d\left(\frac{1}{\cos x}\right)}{dx}$$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = 2 \cdot \frac{\cos x \cdot \cos x - (\sin x) \cdot (-\sin x)}{\cos^2 x}$$

$$- 7(\sec x \cdot \tan x)$$

$$= 2 \cdot \frac{(\cos^2 x + \sin^2 x)}{\cos^2 x} - 7 \sec x \cdot \tan x$$

$$= 2 \sec^2 x - 7 \sec x \cdot \tan x$$

Toppers Village - Class - 11 - Maths

DERIVATIVES

- First Principle of Derivatives

$$\frac{d(f(x))}{dx} = f'(x) = \frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- $(u \pm v)' = u' \pm v'$

- $(u \cdot v)' = u' \cdot v + u \cdot v'$

$$(u \cdot v \cdot w)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$$

- $(K u)' = K \cdot u'$

↑
Constant function

- $\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$

Important Derivatives:

- $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$

- $\frac{d(\sin x)}{dx} = \cos x$

- $\frac{d(\cos x)}{dx} = -\sin x$

- $\frac{d(\tan x)}{dx} = \sec^2 x$

- $\frac{d(\sec x)}{dx} = \sec x \cdot \tan x$

- $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

- $\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cdot \cot x$

$$\cdot \frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\cdot \frac{d(\cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\cdot \frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\cdot \frac{d(\cot^{-1}x)}{dx} = \frac{-1}{1+x^2}$$

$$\cdot \frac{d(\sec^{-1}x)}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\cdot \frac{d(\operatorname{cosec}^{-1}x)}{dx} = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\cdot \frac{d(\log x)}{dx} = \frac{d(\log_e x)}{dx} = \frac{d(\ln(x))}{dx} = \frac{1}{x}$$

Logarithmic
Fm

$e = \text{euler's No.} = 2.718 \dots$

$$\cdot \frac{d(e^x)}{dx} = e^x$$

$$\cdot \frac{d(a^x)}{dx} = a^x \cdot \log_e a$$

Exponential Fm

$$\cdot \frac{d(\text{constant})}{dx} = 0$$

$$\cdot \frac{d\left(\frac{1}{x}\right)}{dx} = -\frac{1}{x^2}$$

$$\cdot \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

$$\cdot \frac{d(x)}{dx} = 1$$

$$\frac{1}{x} = x^{-1}$$

$$\sqrt{x} = x^{\frac{1}{2}}$$

Note:

$$\frac{d(x^n)}{dx} = n \cdot x^{n-1}$$

$$\frac{d(u)}{du} = 1, \quad \frac{d(x)}{dx} = 1, \quad \frac{d(y)}{dy} = 1$$

$$\frac{d(f(u))}{d(u)} = 1$$

$$\boxed{\frac{d(\sin x)}{dx} = \cos x}$$

Also:

$$\frac{d(f(u))^n}{d(u)} = n \cdot (f(u))^{n-1}$$

$$\frac{d(\sin f(u))}{d(u)} = \cos(f(u))$$

e.g.

$$\frac{d(\cos(\sin x))}{d(\sin x)} = -\sin(\sin x)$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

Chain Rule:

function in the function

↓
Composite Function

e.g. $\sqrt{\sin x}$, $\sin(x^2)$

$$\sin(\cos x), \cos(\tan x)$$

$$f(u) \text{ (gen)} = f(g(x))$$

$$(\sin x)^n = \sin^n x$$

e.g. Find the derivative of $\sin(\cos x)$.

e.g. $f(x) = \sin(\cos x)$

Find $f'(x)$.

$$\frac{d(f(x))}{dx} = \frac{d(\sin(\cos x))}{dx}$$

$$= \frac{d(\sin(\cos x))}{d(\cos x)} \cdot \frac{d(\cos x)}{dx}$$

$$= \cos(\cos x) \cdot (-\sin x)$$

e.g. If $f(x) = \sin(\tan(\sqrt{x}))$

find $f'(x)$.

Ans. $\frac{d(f(x))}{dx} = \frac{d(\sin(\tan(\sqrt{x})))}{dx}$

$$= \frac{d(\sin(\tan(\sqrt{x})))}{d(\tan(\sqrt{x}))} \cdot \frac{d(\tan(\sqrt{x}))}{d(\sqrt{x})} \cdot \frac{d(\sqrt{x})}{dx}$$

$$= \cos(\tan(\sqrt{x})) \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

Chain Rule (Fast)

$$\frac{d(f(g(h(x))))}{dx} = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

Composite Funⁿ.



e.g. $f(x) = \sin(\cos x)$

$$f'(x) = \cos(\cos x) \cdot (-\sin x)$$



e.g. $f(x) = \sin(\tan(\sqrt{x}))$

$$f'(x) = \cos(\tan \sqrt{x}) \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$



e.g. $f(x) = \sqrt{\sin(x^2)}$

~~$f(x) = \sqrt{\sin(x^2)}$~~ $= (\sin(x^2))^{\frac{1}{2}}$

~~$x^{\frac{1}{2}}$~~ $x^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2\sqrt{\sin(x^2)}} \cdot \cos(x^2) \cdot 2x$$

$$= \frac{x \cdot \cos(x^2)}{\sqrt{\sin(x^2)}}$$

Toppers Village

Miscellaneous Exercise on Chapter (13)

[Limits & Derivatives]

First Principle of Derivative

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Q.1 (i) $f(x) = -x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-(x+h) - (-x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-\cancel{x} - h + \cancel{x}}{h}$$

$$= -1$$

$$(ii) f(x) = (-x)^{-1} = \frac{1}{(-x)} = -\frac{1}{x}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\left(-\frac{1}{x+h}\right) - \left(-\frac{1}{x}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\left(-\frac{1}{x+h} + \frac{1}{x}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\left(\frac{-x + x+h}{x(x+h)}\right)}{\left(\frac{h}{1}\right)}$$

$$= \lim_{h \rightarrow 0} \frac{1}{x(x+h)}$$

$$= \frac{1}{x(x+0)} = \frac{1}{x^2} \quad \checkmark$$

$$(iii) f(x) = \sin(x+1)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(x+1+h) - \sin(x+1)}{h}$$

$$\boxed{\sin A - \sin B = 2 \sin\left(\frac{A-B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right)}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin\left(\frac{x+1+h - x-1}{2}\right) \cdot \cos\left(\frac{2x+2+h}{2}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin\left(\frac{h}{2}\right) \cdot \cos\left(x+1+\frac{h}{2}\right)}{h} \quad \begin{matrix} \swarrow \\ \text{Indeterminate Form} \\ \searrow \end{matrix} \quad \left(\frac{0}{0} \text{ Form}\right)$$

$$= \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right) \cdot \cos\left(x+1+\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} = 1 \times \cos(x+1+0) = \cos(x+1)$$

Standard Form.

$$\boxed{\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1}$$

$$(iv) f(x) = \cos\left(x - \frac{\pi}{8}\right)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cos\left(x - \frac{\pi}{8} + h\right) - \cos\left(x - \frac{\pi}{8}\right)}{h}$$

$$\boxed{\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \cdot \sin\left(\frac{2x - \frac{2\pi}{8} + h}{2}\right) \cdot \sin\left(\frac{\cancel{x - \frac{\pi}{8}} + h - \cancel{x + \frac{\pi}{8}}}{2}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \cdot \sin\left(x - \frac{\pi}{8} + \frac{h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-\sin\left(x - \frac{\pi}{8} + \frac{h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} = -\sin\left(x - \frac{\pi}{8} + 0\right) \times 1 = -\sin\left(x - \frac{\pi}{8}\right)$$

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Miscellaneous Exercise - Chapter (13)

[Limits & Derivatives]

[Q.2] Derivative $f(x) = x + a$

$$f'(x) = \frac{d(x+a)}{dx}$$

$$= \frac{d(x)}{dx} + \frac{d(a)}{dx}$$

$$= 1 + 0$$

$$= 1$$

[Q.3] $f(x) = [p(x)+q] \cdot \left[\frac{r}{x} + s\right]$

$$f(x) = pr + ps \cdot x + \frac{qr}{x} + qs$$

$$f'(x) = \frac{d\left(pr + ps \cdot x + \frac{qr}{x} + qs\right)}{dx}$$

$$f'(x) = 0 + ps \cdot \frac{d(x)}{dx} + qr \cdot \frac{d\left(\frac{1}{x}\right)}{dx} + 0$$

$$= ps \cdot (1) + qr \cdot \left(-\frac{1}{x^2}\right)$$

$$= ps - \frac{qr}{x^2}$$

$$\frac{d(x^n)}{dx} = n \cdot x^{n-1}$$

$$\begin{aligned} \frac{1}{x} &= x^{-1} \\ &= -1 \cdot x^{-1-1} \\ &= -x^{-2} = -\frac{1}{x^2} \end{aligned}$$

Q. 4 $f(x) = (ax+b).(cx+d)^2$

I - method.

$$f(x) = (ax+b).(cx+d).(cx+d)$$

$$(u \cdot v \cdot w)'$$

$$f'(x) = (a \cdot 1 + 0) \cdot (cx+d) \cdot (cx+d)$$

$$+ (ax+b) \cdot (c \cdot 1 + 0) \cdot (cx+d) \checkmark$$

$$+ (ax+b) \cdot (cx+d) \cdot (c \cdot 1 + 0) \checkmark$$

$$= a \cdot (cx+d)^2 + 2 \cdot c \cdot (ax+b) \cdot (cx+d)$$

✓

$$x^n \rightarrow n \cdot x^{n-1}$$

$$x^2 \rightarrow 2 \cdot x^{2-1} = 2x$$

II - method. (By chain Rule)

$$f(x) = (ax+b).(cx+d)^2$$

$$(u \cdot v)'$$

$$f'(x) = (a+0) \cdot (cx+d)^2$$

$$+ (ax+b) \cdot \frac{d}{dx} (cx+d)^2 \leftarrow$$

$$f'(x) = a \cdot (cx+d)^2 + (ax+b) \cdot 2(cx+d) \cdot (c \cdot 1 + 0)$$

$$f'(x) = a \cdot (cx+d)^2 + 2c(ax+b) \cdot (cx+d)$$

Q.5

$$f(x) = \frac{ax+b}{cx+d} = \frac{u}{v}$$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = \frac{(ax+b)' \cdot (cx+d) - (ax+b) \cdot (cx+d)'}{(cx+d)^2}$$

$$f'(x) = \frac{(a \cdot 1 + 0)(cx+d) - (ax+b) \cdot (c+0)}{(cx+d)^2}$$

$$f'(x) = \frac{\cancel{a}x + ad - \cancel{a}x - bc}{(cx+d)^2}$$

$$f'(x) = \frac{ad - bc}{(cx+d)^2} \quad \checkmark$$

Q.6.

$$f(x) = \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} = \frac{x+1}{x-1}$$

$$f(x) = \frac{x+1}{x-1} = \frac{u}{v}$$

$$f'(x) = \frac{(1+0) \cdot (x-1) - (x+1) \cdot (1-0)}{(x-1)^2}$$

$$= \frac{\cancel{x}-1 - \cancel{x}-1}{(x-1)^2}$$

$$f'(x) = \frac{-2}{(x-1)^2}$$

$$\boxed{Q.7} \quad f(x) = \frac{1}{(ax^2+bx+c)} = \frac{u}{v}$$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = \frac{(0) \cdot (ax^2+bx+c) - 1 \cdot (a \cdot 2 \cdot x + b \cdot 1 + 0)}{(ax^2+bx+c)^2}$$

$$(x^2 \rightarrow x^n) \quad \frac{d(x^n)}{dx} = n \cdot x^{n-1}$$

$$f'(x) = \frac{0 - 2ax - b}{(ax^2+bx+c)^2}$$

$$f'(x) = \frac{-(2ax+b)}{(ax^2+bx+c)^2}$$

$$\boxed{Q.8} \quad f(x) = \frac{ax+b}{px^2+qx+r} = \frac{u}{v}$$

$$f'(x) = \frac{(a+0) \cdot (px^2+qx+r) - (ax+b) \cdot (p \cdot 2x + q + 0)}{(px^2+qx+r)^2}$$

$$\Rightarrow f'(x) = \frac{a(px^2+qx+r) - (ax+b)(2px+q)}{(px^2+qx+r)^2}$$

$$\Rightarrow f'(x) = \frac{apx^2 + aqx + ar - 2apx^2 - axq - 2bpx - bq}{(px^2+qx+r)^2}$$

$$\Rightarrow f'(x) = \frac{-apx^2 - 2bpx + ar - bq}{(px^2+qx+r)^2}$$

Q.9 $f(x) = \frac{px^2 + qx + r}{ax + b} = \frac{u}{v}$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = \frac{(2px + q + 0) \cdot (ax + b) - (px^2 + qx + r) \cdot (a + 0)}{(ax + b)^2}$$

$$f'(x) = \frac{apx^2 + 2bpx - ar + bq}{(ax + b)^2}$$

$$\frac{d(x^n)}{dx} = n \cdot x^{n-1}$$

Q.10 $f(x) = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$

$$f(x) = a \cdot (x^{-4}) - b \cdot (x^{-2}) + (\cos x)$$

$$f'(x) = a \cdot (-4 \cdot x^{-4-1}) - b \cdot (-2 \cdot x^{-2-1}) + (-\sin x)$$

$$f'(x) = -\frac{4a}{x^5} + \frac{2b}{x^3} - \sin x$$

Q.11

$$f(x) = 4\sqrt{x} - 2$$

$$f(x) = 4(x)^{\frac{1}{2}} - 2$$

$$f'(x) = 4 \cdot \frac{1}{2} \cdot x^{\frac{1}{2}-1} - 0$$

$$f'(x) = 2 \cdot x^{-\frac{1}{2}}$$

$$f'(x) = \frac{2}{x^{\frac{1}{2}}}$$

$$f'(x) = \frac{2}{\sqrt{x}}$$

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Miscellaneous Exercise - Chapter (13)

Limits and Derivatives

[Q.12] $f(x) = (ax+b)^n$

$$f'(x) = \frac{d(ax+b)^n}{dx}$$

$$\frac{d(x^n)}{dx} = n \cdot x^{n-1}$$

Chain Rule

$$= \frac{d(ax+b)^n}{d(ax+b)} \cdot \frac{d(ax+b)}{dx}$$

$$= n \cdot (ax+b)^{n-1} \cdot (a \cdot 1 + 0)$$

$$= na(ax+b)^{n-1}$$

II - method: (fast) - Chain Rule.

$$f(x) = (ax+b)^n$$

$$\Rightarrow f'(x) = n \cdot (ax+b)^{n-1} \cdot (a \cdot 1 + 0) = na(ax+b)^{n-1}$$

[Q.13]

$$f(x) = (ax+b)^n \cdot (cx+d)^m$$

$$(u \cdot v)' = u'v + u \cdot v'$$

$$f'(x) = \frac{d(ax+b)^n}{dx} \cdot (cx+d)^m + (ax+b)^n \cdot \frac{d(cx+d)^m}{dx}$$

$$= n(ax+b)^{n-1} \cdot (a \cdot 1 + 0) \cdot (cx+d)^m + (ax+b)^n \cdot m \cdot (cx+d)^{m-1} \cdot (c \cdot 1 + 0)$$

$$= na(ax+b)^{n-1} \cdot (cx+d)^m + mc(ax+b)^n \cdot (cx+d)^{m-1}$$

$$= (ax+b)^{n-1} \cdot (cx+d)^{m-1} \cdot \left\{ na(cx+d) + mc(ax+b) \right\}$$

Q.14 $f(x) = \sin(x+a)$
 $\rightarrow$ $\rightarrow$ Chain Rule

$$\Rightarrow f'(x) = \cos(x+a) \cdot (1+0)$$

$$\Rightarrow f'(x) = \cos(x+a) \quad \checkmark$$

Q.15 $f(x) = \operatorname{cosec} x \cdot \cot x$
 $(u \cdot v)' = u' \cdot v + u \cdot v'$

$$f'(x) = \frac{d(\operatorname{cosec} x \cdot \cot x)}{dx}$$

$$= \frac{d(\operatorname{cosec} x)}{dx} \cdot \cot x + \operatorname{cosec} x \cdot \frac{d(\cot x)}{dx}$$

$$= (-\operatorname{cosec} x \cdot \cot x) \cdot \cot x + \operatorname{cosec} x \cdot (-\operatorname{cosec}^2 x)$$

$$= -\operatorname{cosec} x \cdot \cot^2 x - \operatorname{cosec}^3 x$$

Q.16 $f(x) = \frac{\cos x}{1 + \sin x} = \frac{u}{v}$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = \frac{(-\sin x) \cdot (1 + \sin x) - \cos x \cdot (0 + \cos x)}{(1 + \sin x)^2}$$

$$f'(x) = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$$

$$f'(x) = - \left(\frac{+\sin x + (1)}{(1 + \sin x)^2} \right)$$

$$= - \frac{(1 + \sin x)}{(1 + \sin x)^2}$$

$$= - \frac{1}{(1 + \sin x)} \quad \checkmark$$

Q.17 $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x} = \frac{u}{v}$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = \frac{(\cos x - \sin x) \cdot (\sin x - \cos x) - (\sin x + \cos x) \cdot (\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$= \frac{- (\sin x - \cos x)^2 - (\sin x + \cos x)^2}{(\sin x - \cos x)^2}$$

$$= - \left\{ \frac{\sin^2 x + \cos^2 x - 2 \sin x \cos x + \sin^2 x + \cos^2 x + 2 \sin x \cos x}{(\sin x - \cos x)^2} \right\}$$

$$= - \frac{2}{(\sin x - \cos x)^2}$$

Q.18 $f(x) = \frac{\sec x - 1}{\sec x + 1} = \frac{u}{v}$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = \frac{[\sec x \cdot \tan x - 0] \cdot (\sec x + 1) - (\sec x - 1) \cdot [\sec x \cdot \tan x + 0]}{(\sec x + 1)^2}$$

$$= \frac{\cancel{\sec^2 x \cdot \tan x} + \sec x \cdot \tan x - \cancel{\sec^2 x \cdot \tan x} + \sec x \cdot \tan x}{(\sec x + 1)^2}$$

$$= \frac{2 \sec x \cdot \tan x}{(\sec x + 1)^2}$$

Miscellaneous Exercise - Chapter (13)

Limits & Derivatives

Q.19 $f(x) = \sin^n x$

$f(x) = (\sin x)^n$

Chain Rule

$f'(x) = \frac{d(\sin x)^n}{dx}$

$= n \cdot (\sin x)^{n-1} \times \cos x$

$= n \cdot \cos x \cdot (\sin x)^{n-1}$

Q.20

$f(x) = \frac{a + b \sin x}{c + d \cos x}$

$\left(\frac{u}{v}\right)'$

$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$

$f'(x) = \frac{\frac{d}{dx}(a + b \sin x) \cdot (c + d \cos x) - \frac{d}{dx}(c + d \cos x) \cdot (a + b \sin x)}{(c + d \cos x)^2}$

$= \frac{(0 + b \cos x) \cdot (c + d \cos x) - [0 + d(-\sin x)] \cdot (a + b \sin x)}{(c + d \cos x)^2}$

$= \frac{bc \cos x + bd \cos^2 x + ad \sin x + bd \sin^2 x}{(c + d \cos x)^2}$

$= \frac{bc \cos x + ad \sin x + bd(1)}{(c + d \cos x)^2}$

Q.21

$$f(x) = \frac{\sin(x+a)}{\cos x} \quad \left(\frac{u}{v}\right)$$

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = \frac{\frac{d(\sin(x+a))}{dx} \cdot \cos x - \sin(x+a) \cdot \frac{d(\cos x)}{dx}}{(\cos x)^2}$$

$$= \frac{\cos(x+a) \cdot (1+0) \cdot \cos x - \sin(x+a) \cdot (-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos(x+a) \cdot \cos x + \sin(x+a) \cdot \sin x}{(\cos x)^2}$$

$$\begin{aligned} &\cos A \cdot \cos B \\ &+ \sin A \cdot \sin B \\ &= \cos(A-B) \end{aligned}$$

$$= \frac{\cos(x+a-x)}{(\cos x)^2} = \frac{\cos a}{(\cos x)^2} \quad \checkmark$$

Q.22 $f(x) = \overset{u}{x^4} \cdot \overset{v}{(5\sin x - 3\cos x)}$

$(u \cdot v)' = u'v + u \cdot v'$

$$f'(x) = \frac{d(x^4)}{dx} \cdot (5\sin x - 3\cos x) + x^4 \cdot \frac{d(5\sin x - 3\cos x)}{dx}$$

$$f'(x) = 4 \cdot x^3 \cdot (5\sin x - 3\cos x) + x^4 \cdot (5 \cdot \cos x + 3 \sin x)$$

$$= x^3 \cdot \left\{ \begin{array}{l} 20 \sin x - 12 \cos x \\ + 5x \cos x + 3x \sin x \end{array} \right\}$$

✓

Q.23 $f(x) = (x^2 + 1) \cdot \cos x$

$(u \cdot v)' = u'v + u \cdot v'$

$$f'(x) = (2 \cdot x + 0) \cdot \cos x + (x^2 + 1) \cdot (-\sin x)$$

$$= \underline{2x \cos x} - \underline{\sin x} \cdot \underline{(x^2 + 1)}$$

Q.24

$$f(x) = (ax^2 + \sin x) \cdot (p + q \cos x)$$

$$\boxed{(\underbrace{u} \cdot \underbrace{v})' = \underbrace{u'} \cdot \underbrace{v} + \underbrace{u} \cdot \underbrace{v'}} \quad \begin{matrix} \downarrow & \downarrow \end{matrix}$$

$$\begin{aligned} f'(x) &= (a \cdot 2x + \cos x) \cdot (p + q \cos x) \\ &\quad + (ax^2 + \sin x) \cdot (0 + q(-\sin x)) \end{aligned}$$

$$\begin{aligned} f'(x) &= (2ax + \cos x) \cdot (p + q \cos x) \\ &\quad - q \sin x \cdot (ax^2 + \sin x) \end{aligned}$$

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Miscellaneous Exercise - Chapter (3)

Limits & Derivatives

Q.25 $f(x) = (x + \cos x)(x - \tan x)$
 $(u \cdot v)' = u' \cdot v + u \cdot v'$

$$f'(x) = \frac{d}{dx}(x + \cos x) \cdot (x - \tan x) + (x + \cos x) \cdot \frac{d}{dx}(x - \tan x)$$

$$= (1 - \sin x) \cdot (x - \tan x) + (x + \cos x) \cdot (1 - \sec^2 x)$$

Q.26 $f(x) = \frac{4x + 5 \sin x}{3x + 7 \cos x} \left(\frac{u}{v} \right)$

$$\left(\frac{u}{v} \right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$-(4x + 5 \sin x) \cdot (3 - 7 \sin x)$

$$f'(x) = \frac{(4 + 5 \cos x) \cdot (3x + 7 \cos x)}{(3x + 7 \cos x)^2}$$

$$= \frac{(4 + 5 \cos x) \cdot (3x + 7 \cos x) - (4x + 5 \sin x) \cdot (3 - 7 \sin x)}{(3x + 7 \cos x)^2}$$

$$= \frac{12x + 28 \cos x + 15x \cos x + 35 \cos^2 x - 12x + 28x \sin x - 15 \sin x + 35 \sin^2 x}{(3x + 7 \cos x)^2}$$

$$= \frac{28 \cos x + 15x \cos x + 28x \sin x - 15 \sin x + 35}{(3x + 7 \cos x)^2}$$

$$\boxed{\text{Q.27}} \quad f(x) = \frac{x^2 \cdot \cos \frac{\pi}{4}}{\sin x} \quad \left(\frac{u}{v} \right)$$

$$\left(\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} = \underline{\text{constant}} \right)$$

$$\frac{d f(x)}{dx} = \frac{d \left(\frac{x^2 \cdot \cos \frac{\pi}{4}}{\sin x} \right)}{dx}$$

$$= \cos \frac{\pi}{4} \cdot \frac{d \left[\frac{x^2}{\sin x} \right]}{dx} \quad \left(\frac{u}{v} \right)$$

$$\left(\frac{u}{v} \right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$= \cos \frac{\pi}{4} \cdot \frac{(2x) \cdot \sin x - x^2 \cdot (\cos x)}{\sin^2 x}$$

$$= \cos \frac{\pi}{4} \cdot \left(\frac{2x \sin x - x^2 \cos x}{\sin^2 x} \right)$$

$$\boxed{\text{Q.28}} \quad f(x) = \frac{x}{1 + \tan x} \quad \left(\frac{u}{v} \right)$$

$$f'(x) = \frac{(1) \cdot (1 + \tan x) - (x) \cdot (0 + \sec^2 x)}{(1 + \tan x)^2}$$

$$f'(x) = \frac{1 + \tan x - x \sec^2 x}{(1 + \tan x)^2} \quad \checkmark$$

$$\boxed{\text{Q.29}} \quad f(x) = (x + \sec x) \cdot (x - \tan x)$$

$$\downarrow \quad \downarrow$$

$$(u \cdot v)'$$

$$= u' \cdot v + u \cdot v'$$

$$f'(x) = (1 + \sec x \cdot \tan x) \cdot (x - \tan x) + (x + \sec x) \cdot (1 - \sec^2 x)$$

30) $f(x) = \frac{x}{\sin^n x} \left(\frac{u}{v} \right)$

$$\left(\frac{u}{v} \right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$f'(x) = \frac{d \left(\frac{x}{\sin^n x} \right)}{dx}$$

$$= \frac{\frac{d(x)}{dx} \sin^n x - x \cdot \frac{d(\sin^n x)}{dx}}{(\sin^n x)^2}$$

$$= \frac{1 \cdot \sin^n x - x \cdot (n \cdot \cos x \cdot (\sin x)^{n-1})}{(\sin^n x)^2}$$

$$= \frac{\sin^n x - nx \cdot \cos x \cdot \sin^{n-1} x}{\sin^{2n} x}$$

$$\sin^n x = (\sin x)^n$$

Chain Rule

$$\frac{d(\sin x)^n}{dx} = n \cdot (\sin x)^{n-1} \cdot \cos(x) \cdot 1$$

$$= \underline{\underline{n \cdot \cos x \cdot (\sin x)^{n-1}}}$$

~~$x^n \rightarrow n \cdot x^{n-1}$~~

$$= \frac{\cancel{\sin^{n-1} x} (\sin x - nx \cdot \cos x)}{(\cancel{\sin x})^{n-1} (\sin x)^{n+1}}$$

$$= \frac{\sin x - nx \cdot \cos x}{(\sin x)^{n+1}}$$

All Useful Links:

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** www.toppersvillage.com
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** www.facebook.com/ToppersVillage/
- **Facebook Group:** <https://www.facebook.com/groups/ToppersVillage>